CCD TRANSFORMATION EQUATIONS
FOR USE WITH SINGLE IMAGE (DIFFERENTIAL) PHOTOMETERY

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Abstract

CCD photometry requires that corrections be made for a specific user's unique spectral response for blue, visible, red and infra-red filters. A system's calibration is unique princiapally due to filter pass band characteristics, the telescope's spectral response, the CCD's spectral response and also atmospheric extinction properties that depend on elevation angle in a way that changes with time. The "CCD Transformation Equations" are intended to remove most of these effects, allowing for the combining of multi-color photometric observations contributed by many observers.

The specific set of equations to be used depends on which of several possible filter combinations are used. Whenever at least two filters are employed, such as BV, VR or RI, transformation equation corrections can be applied. The most common set in use is the three color combination BVR. For this filter set the following transformation equations to be used are (in the order of use):

$(V_s - R_s) = (V_c - R_c) + T_{vr} * [(v_s - r_s) - (v_c - r_c)]$
$V_s = v_s + (V_c - v_c) + T_v * [(V_s - R_s) - (V_c - R_c)]$, *using the solution for $(V_s - R_s)$ in the above line*
$R_s = V_s - (V_s - R_s)$, *using V_s from the line above, and $(V_s - R_s)$ from the first line*
$B_s = V_s + (B_c - V_c) + T_{bv} * [(b_s - v_s) - (b_c - v_c)]$

where:

V_c = *known V-band magnitude of comparison star,*
R_c = *known R-band magnitude of comparison star*,

v_s = *instrumental (i.e., measured and uncorrected) V-band magnitude of star of interest,*
v_c = *instrumental V-band magnitude of known comparison star,*

r_s = *instrumental R-band magnitude of star of interest,*
r_c = *instrumental R-band magnitude of comparison star,*

assuming the following coefficient definitions (unique to an observing system and usually unchanging):

Tv = slope of (V-v) plotted versus (V-R)
Tvr = reciprocal of slope of (v-r) plotted versus (V-R)
Tbv = reciprocal of slope of (b-v) plotted versus (B-V)

The last group of three transformation coefficients should be evaluated once or twice per year (or whenever the observing system changes), and they are to be evaluated using a field of known standard stars (such as M67). This web page describes the derivation of the above equations, as well as the equations that are to be used for all other filter combinations. Examples are given for deriving the coefficients, and reducing observations with the BVR set. An alternative procedure for correcting BVR observations (simpler and more accurate) is described on another web page, linked to at the end of this web page.

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I. Introduction.

This web page is meant for new astro-CCD observers wishing to make photometric quality measurements of the magnitude of stellar objects

using BVRI filters.

When I started doing this it was difficult to locate information on the internet, or in books for amateur astronomers, that explained the procedure for correcting color filter observations for errors caused by slight differences between an observer's "system spectral response" for each filter and a reference standard spectral response. A commonly used correction procedure makes use of "CCD Transformation Equations" which require that each observer determine **coefficients** used by the transformation equations. Each observing system has a unique set of coefficient values, and these values should be determined about once or twice per year to monitor any changes in filter or CCD properties. The purpose of this web page is to present a derivation of the the procedure for determining the transformation equation coefficients, the procedure for using the transformation equations to correct a set of observations, and the rationale for this entire procedure.

The AAVSO (American Association of Variable Star Observers) has a web page explaining the use of CCDs for photometry, and it also contains a description of CCD Transformation Equations. This is a great starting place for any observer wanting to understand how to perform quality CCD photometry for submission to the AAVSO. That page is at AAVSO CCD Photometry Manual.

Personally, I think this entire procedure of using CCD Transformation Equations is ridiculously complicated and totally unnecessary, since there is a simpler and more accurate way of achieving transformations, assuming you know how to use a spreadsheet program. This procedure is described in the Differential Photometry section of All-Sky Photometry. More on this at the end of this tedious web page.

## II. The Reasons Magnitude Corrections Are Necessary

This section illustrates the way typical differences in filters spectral response and CCD spectral response contribute to errors in the measurements of star magnitudes. It makes the case for the practice of performing CCD transformation equation corrections before submission to AASVO. The following sections show how to make those corrections.

In order to compare observations of the brightness of a given object made by different observers it is necessary to correct for differences in the spectral response of each oberver's system and observing situation. The term "observer's system" refers to the CCD, filters, and telescope. The term "observing situation" refers to the atmosphere through which the observations are made. Both the hardware and the atmosphere can affect the spectral response associated with an observation using a specific filter.

The concept of "spectral response" is important throughout all that is dealt with here, so let's deal with that first. Consider a single observation (or integration) of a field of interest using a single filter. The term"spectral response" refers to the probability that photons of light having different energies (wavelengths) will successfully pass through the atmosphere (without being scattered or absorbed) and pass through the telescope and filter and then be registered by the CCD at some pixel location. This probability versus wavelength, called spectral response, varies with photon wavelength, ranging from zero at all short wavelengths, to maybe 20% (as described below) near the center of the filter's response function, and back to zero for all longer wavelengths. The spectral response will be a smooth function, having steep slopes on both the short-wavelength cut-on and long wavelength cut-off sides of the response function. The entire journey of a photon through the atmosphere, the telescope, the filter, and it's interaction with the CCD chip, where it hopefully will dislodge an electron that will later be collected by the CCD electronics when the integration has finished, can be summarized by "probability versus wavelength" functions, described next.

Atmospheric Absorption and Scattering

The first obstacle on a photon's journey to the observer's image file is the atmosphere.
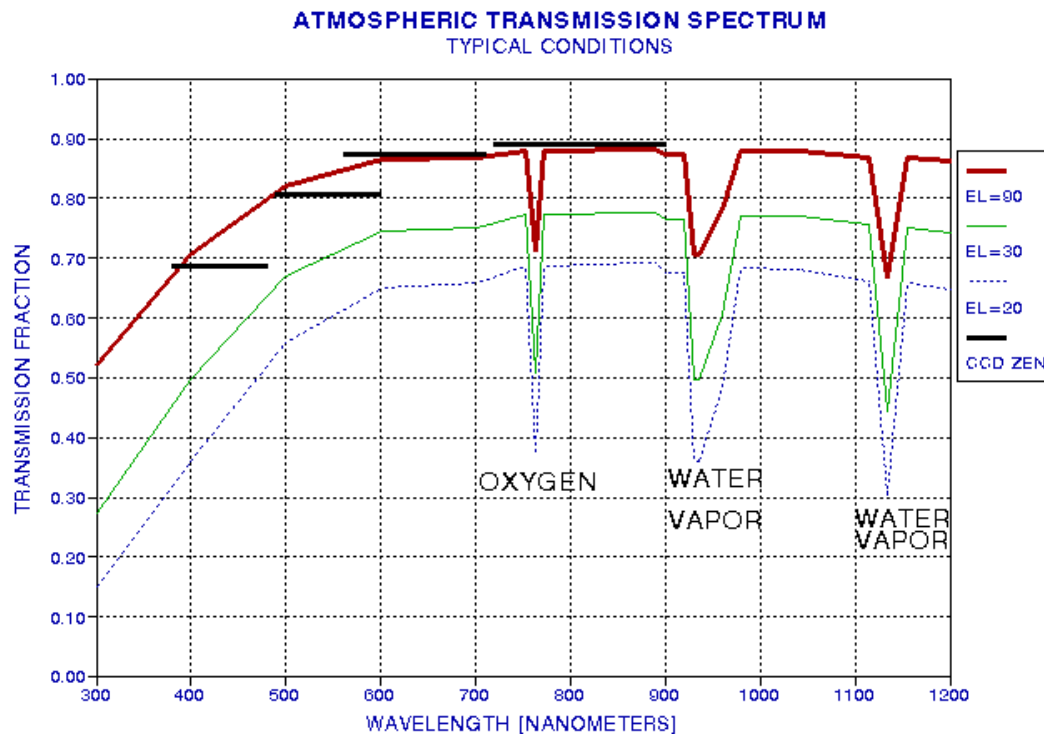
Figure 1. *Atmospheric transmission versus wavelength for typical conditions (water vapor burden of 2 cm, few aerosols), for three elevation angles (based on measurements by the author in 1990, at JPL, Pasadena, CA). Three absorption features are evident: a narrow feature at 763 nm, caused by oxygen molecules, and regions at 930 and 1135 nm caused by water vapor molecules. Four thick black horizontal lines show zenith transparency based on measurements made (by the author) with a CCD/filter wheel/telescope for typical clear sky conditions on another date and at another location (2002.04.29, Santa Barbara,CA, SBIG ST-8E, Schuler filters B, V, R and I).*

This figure can be used to state, for example, that photons of wavelength 500 nm (nanometers) coming in from a 30 degree elevation angle have a 67% probability of reaching the ground. The same photons coming from lower elevation angles have a lower probability of reaching the ground. Due to absorption by water vapor molecules, at 940 nm the probability of photons reaching the ground is only about 40% for the 20 degree elevation angle path. During cloudless conditions most photons make it to ground level without being absorbed or scattered.

The depth of the three absorption "features" are not the same for all observers. For example, the 763 nm oxygen feature is altitude dependent, being about 60% of its surface value at Mauna Kea's altitude (13,800 feet). For most amateurs this feature can be disregarded since it is has such a narrow spectral width and its site dependence is small. Within the two water vapor absorption regions, however, changes in absorption can be dramatic. The number of water vapor molecules per unit volume decreases rapidly with altitude. At upwind coastal sites (such as California's west coast) most of the overhead water vapor is confined to the lowest 2000 feet; and above this marine "planetary boundary layer" is dry continental air, with quite small overhead water vapor contents. A sea level site may have twice the overhead water vpaor "burden" as a site at the top of a 2000-foot coastal mountain range. At Mauna Kea, for example, the water vapor absorption features will be miniscule (I once measured a value that was a mere 3% of a corresponding sea level value). Water vapor burdens vary with season at mid-latitudes (about 3 to 1) but do not vary with season in the tropics (where average water vapor burdens are three times greater than at mid-latitudes). At mid-latitudes there can be 2 to 1 variations from week to week, due to weather systems moving through. The absorption spectrum in Fig. 1 is just a guide to what's happening to photons on their journey to ground level.

Filter Pass Bands

Assuming the observer is using a reflector telescope, or a Schmidt-Cassegrain with small losses in the front glass corrector, the photon that makes it to ground level has a lossless path through the telescope to the filter. For observers using a refractor telescope, there may be losses in the objective lens due to reflections and absorptions. For a good objective, though, these losses will be small. The remainder of this section deals with what happens to ground-level photons that reach the filter.

There are two commonly used UBVRI filter response "standards" in use, going by the names Cousins/Bessell and Johnson. Most amateurs use filters adhering to the Johnson response shape. The two systems are essentially the same for UBV, and differ slightly for the R and I filters. Observations made with one filter type can be converted to the other using the CCD transformation equations, so it would be wrong to say that one is better than the other. The choice of one system over the other is less important than a proper use of either one (as Optec forcefully states

on their web page). Even filters from different manufacturers differ slightly from each other. The following figure shows a typical filter response for BVRI filters made by Schuler.
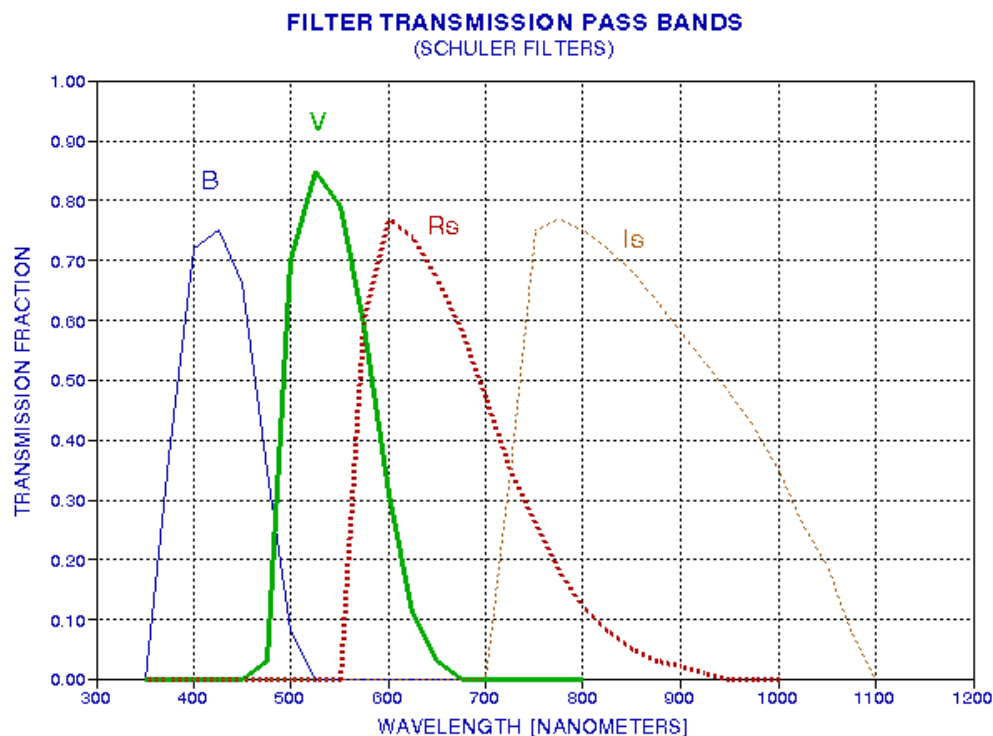

Figure 2. *Typical BVRI filter spectral response (i.e., band pass).*

Considering those 500 nm photons coming in from a 30 degree elevation, for which only 67% make it to ground level, they may have another 70% probability of passing through the V-filter, for example. In other words, only 47% of photons at the top of the atmosphere and coming in at a 30 degree elevation angle make it to the surface of the CCD chip.

CCD Chip Quantum Efficiency

Photons that make it through the atmosphere and filter still must reach the CCD chip if they are to register with the observer's image. There's a matter of cover plates, protecting the chip and preventing water vapor condensation, which is a minor obstacle for a photon's journey. The real challenge for photons is to deposit its energy within a pixel part of the CCD chip and dislodge an electron, setting it free to roam where it can be collected and later produce a voltage associated with the totality of electrons collected at that pixel location. The fraction of photons incident upon the CCD that can produce electrons in a collection "well" is the CCD's quantum efficiency. The quantum efficiency versus wavelength for a commonly used CCD chip is shown in the next figure.
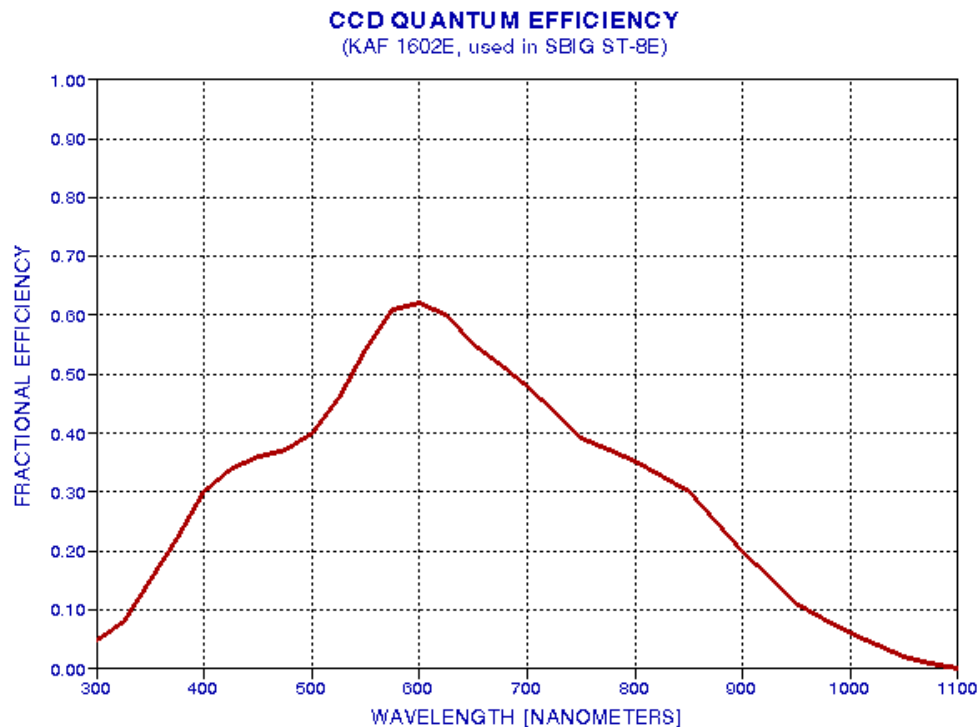
Figure 3. *Fraction of photons incident upon chip that free electrons for later collection (KAF 1602E chip, used in the popular SBIG ST-8E CCD imager).*

Considering again 500 nm photons, of those that reach a typical CCD chip, such as the one used in SBIG's ST-8E, only 40% dislodge an electron for later collection and measurement. For the V-filter, therefore, only 19% of those photons at the top of the atmosphere, coming in at 30 degrees elevation angle, actually get "counted" during an integration under typical clear weather conditions. This number is the product of three transmission functions given in the above three figures. Each filter has associated with it a total transmission probability, and it depends upon not only the filter characteristics, but also upon the atmosphere and the CCD properties. For the system used in this example, the following figure shows the spectral response for photons arriving at a 30 degree elevation angle, under typical weather conditions, going through Schuler filters and being detected by the KAF 1602E CCD chip.

Spectral Response Due to All Sourcse of Photon Loss

The following figure shows the fraction of photons starting at the top of the atmosphere that can be expected to contribute to a star's image for a typical atmosphere conditions, using typical filters and a commonly used CCD.
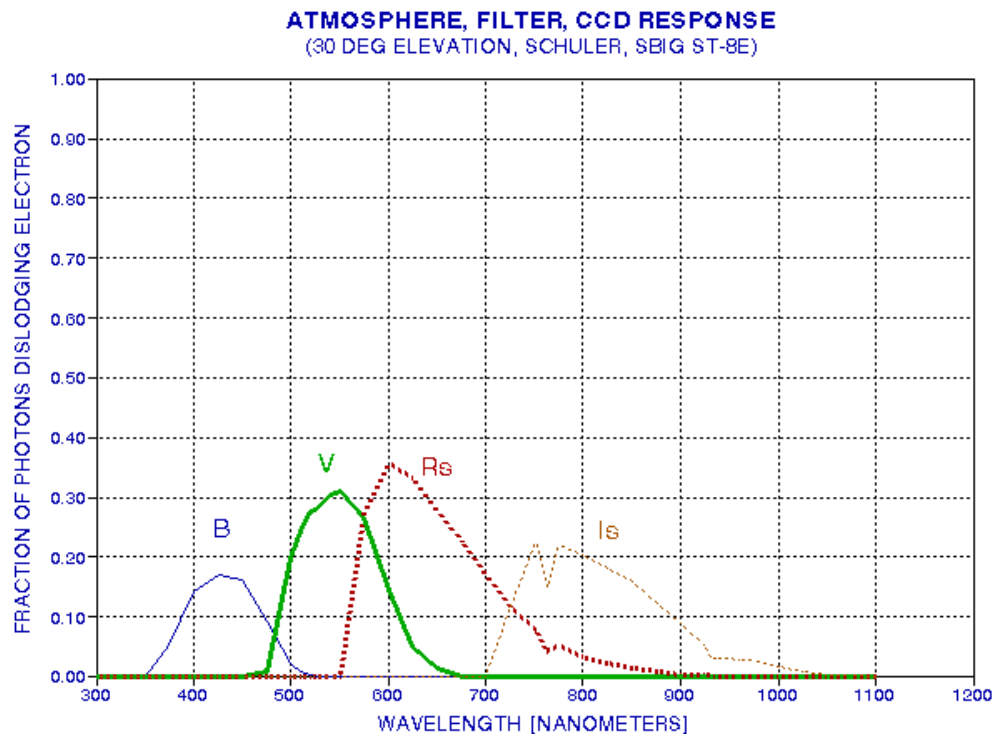
Figure 4. *Response of entire "atmosphere/filter/CCD system" for typical water vapor burden, few aerosols,30 degree elevation angle, Schuler filters and SBIG ST-8E CCD (KAF 1602E chip).*

The reader may now understand how it happens that different observers can have different system spectral responses for their specific systems and atmospheric conditions. Two observers may be making measurements ar the same time from different locations and using different filters and CCD imagers, and unless care is taken to convert their measurements to a "standard system" their reported magnitudes would differ. The magnitude differences will depend upon the "color" of the star under observation, as described in the next section.

Different Observers Have Different Pass Bands

To illustrate the fact that different observers can have different pass bands when they're both making B-filter measurements, let's consider two observers working side-by-side but using different filters and CCD. For example, before I purchased a SBIG ST-8E with Schuler filters, I used a Meade 416XTE CCD with their RGB filter set. The Meade B filter was intended for RGB color image creation, not for photometry. Since the filters weren't designed for photometry (as Meade acknowledges) they will require large corrections during the process of converting observations made with them to a standard system. For the purpose of this discussion, illustrating the concepts of filter differences, the Meade 616 filters provide are suitable example of the need to be careful. The next figure shows the "atmosphere/B-filter/CCD" spectral responses.for the two systems under consideration.
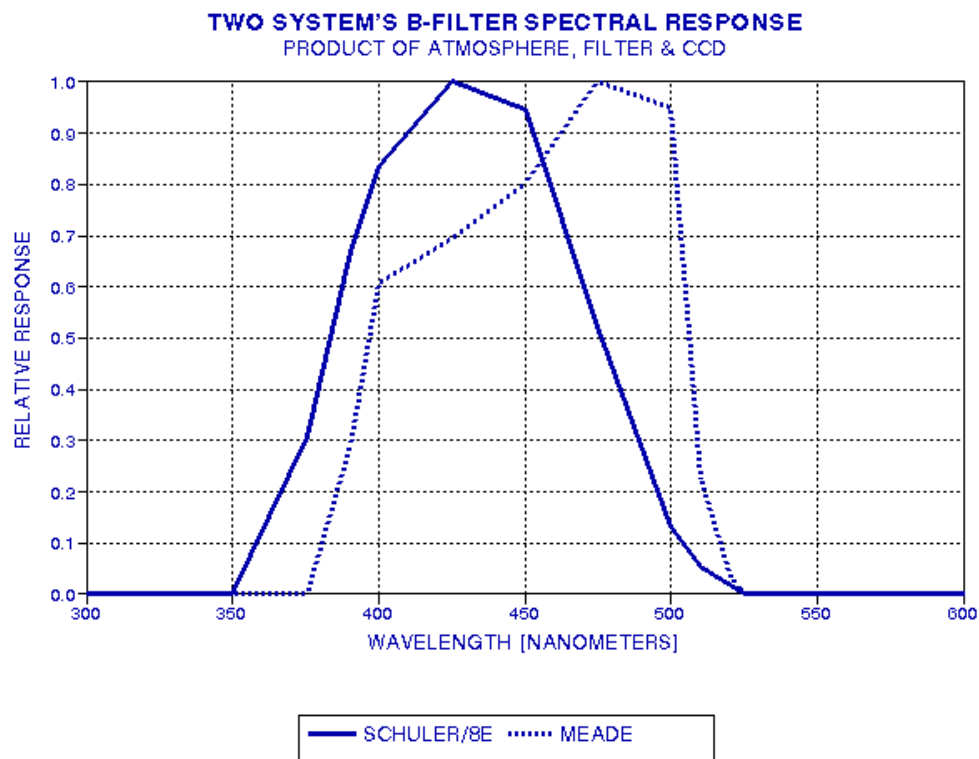
Figure 5. *Spectral response of different systems. The solid trace consists of a Schuler Bu filter, intended for photometry, and a SBIG ST-8E CCD, whereas the dotted trace is for a Meade B-filter and 416XTE CCD. The response for both systems corresponds to observing at an elevation angle of 30 degrees in a typical, clean atmosphere (2 cm precipitable water vapor). Both response traces are normalized to one at their peak response wavelength.*

The Meade system has a spectral response that is shifted to longer wavelengths compared to the Schuler/SBIG ST-8E system. This shift may not seem like much, but consider how important it can be when observing stars with a spectral output that usually is falling off at shorter wavelengths throughout the wavelength region of these filter pass bands. The next figure shows a typical star's brightness versus wavelength.
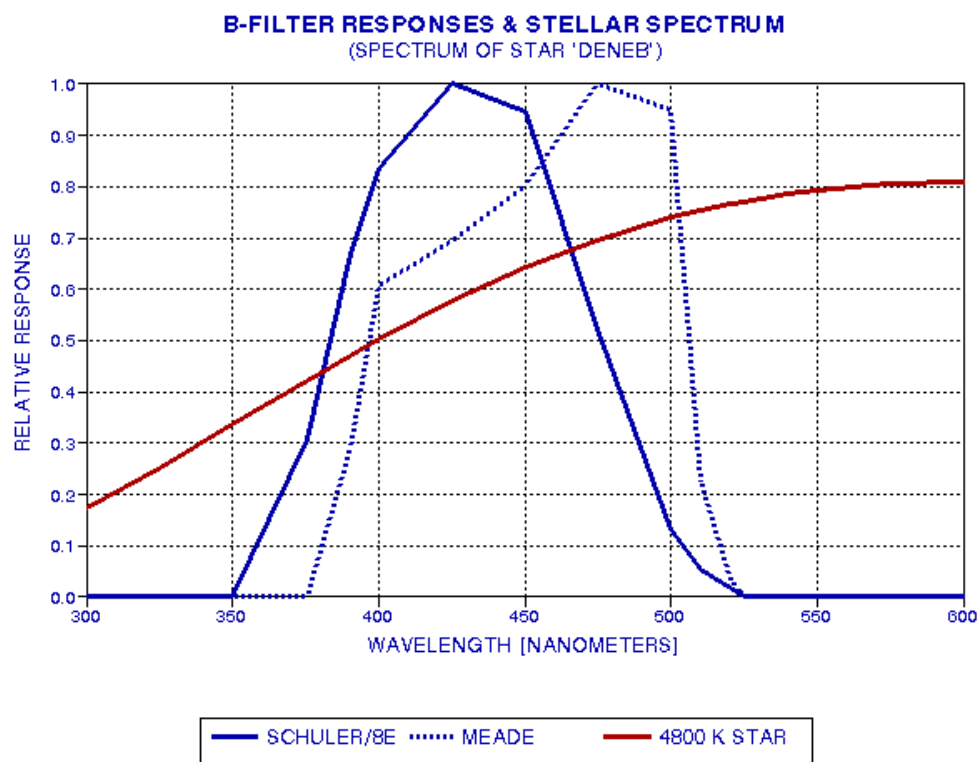
Figure 6. *Typical stellar spectrum (red trace) in relation to spectral response of two different observing systems configured for B-band observing.*

III. Derivation of CCD Transformation Correction Equations - 4 Color Example

The remainder of this web page pertains to observers who will doing "single image photometry" (also called "differential photometry"). For the observer interested in "all sky photometry" I would like to suggest that you read through All Sky Photometry Tutorial, as well as other web pages, articles, and books, and then decide if the "all sky" version is worth a try. Until then, feel a warm welcome to the forgiving world of "single image photometry" - and read on.

The following derivation is built up from simple cases, starting with approximations that illustrate the underlying concepts. Rigourous refinments are applied where needed.

The previous section should have convinced the reader that errors due to filter/CCD differences will vary with a star's "color." Hot stars are blue, cold stars are red. This just means that the "blackbody" distribution of radiant energy varies with wavelength differently, such that the slope of brightness versus wavelength across the region covered by a pair of filters is different for different stars. Comparison stars will have different temperatures, and hence they will have different spectral slopes for any pair of filters under consideration. The target object, such as a supernova, will certainly be have a different spectral shape than the comparisons stars (being much redder, usually).

To begin, let's consider 2-color photometry, using R and I filters (red and infra-red). Later, we'll generalize this to the rest of the filter combinations.

Deriving Something Called T_{ri}

Given that errors in measured magnitude can be expected to correlate with star color, it is natural to wonder about the shape of scatter diagrams that will show this color dependence of measured magnitudes. Consider the case where a field of stars with known magnitudes are observed in a single CCD image. Measured R and I magnitudes are referred to as "r" and "i" whereas known magnitudes are referred to as "R" and "I". Two scatter plots will be found useful: 1) "R-I" versus "r-i" and 2) "R-r" versus "R-I". This is illustrated in the following figures.
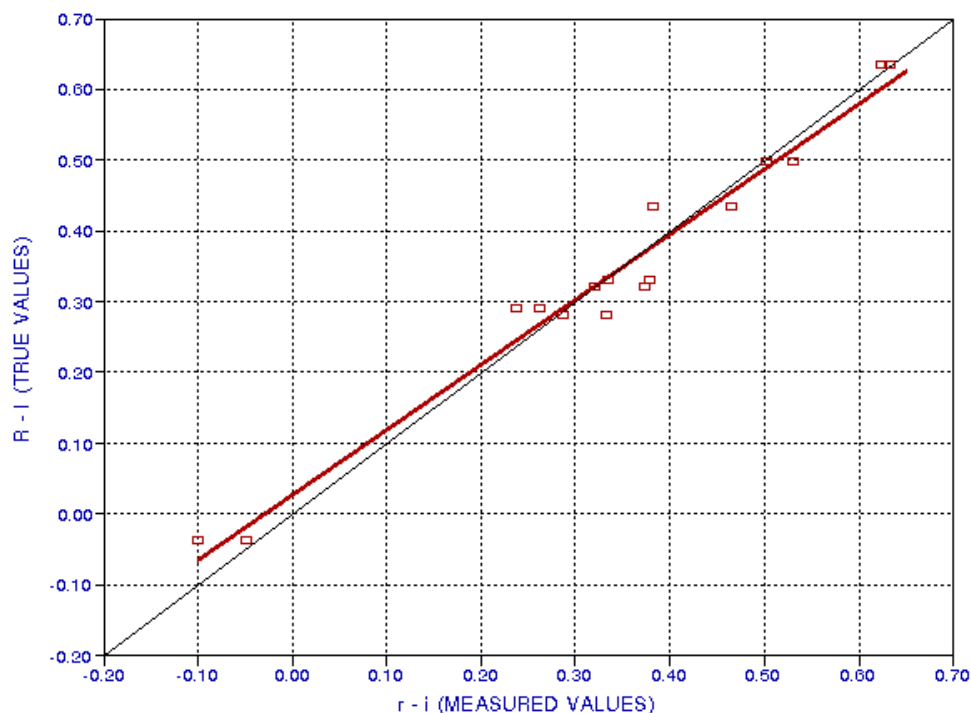
Figure 7. *Actual data (SBIG ST-8E and Schuler filters) for standard comparison stars in M67, showing a plot of known "R-I" versus measured "r-i." (Note: This scatter diagram is not a version that is used in deriving CCD transformation equation coefficients; it is presented here simply to illustrate a concept.)*

The scatter plot in Fig. 7 illustrates that there appears to be a linear relation between true and measured star color "red minus infra-red." Consider this linear relation to be described by the following equation:

$$R-I = C_0 + C_1 * (r-i) \qquad\qquad\qquad\qquad\qquad \text{(Eqn. 1)}$$

In other words, from an analysis of measured red and infra-red magnitudes of a set of standard stars it appears possible to convert any measured r-i star color to a corrected R-I star color. All that's needed are the two coefficients C_0 and C_1 in order to make the conversion. For example, if an object of interest, a supernova say, has a measured r-i of +0.10, then it seems safe to conclude that it has a true R-I of +0.12 (just by simply reading a Y-axis value off the fitted line at the X-axis value of +0.10).

Notice that we're using r and i magnitudes that haven't been calibrated yet. At this stage of analysis, the measured r and i magnitudes can have any calibration **offset** imaginable, and the procedure for determining a "corrected" R-I will not be affected.

The above description is just a small first step in understanding the concepts underlying CCD transformation equations. Before proceeding further, let's back-up a little and modify what we've done to conform with a fundamental principle pertaining to the handling of measurements. Notice that the scatter plot in Fig. 7 has measured values plotted as the abscissa (X-axis) and the true values as the ordinate (Y-axis). If error bars were used they'd be horizontal, which is kind of unusual. It's unusual because whenever measured values are "fitted" they must be treated as the dependent variable, reserving the true values as the independent variable. It is customary to represent this relationship by plotting measured quantities along the Y-axis versus known quantities along the X-axis. Thus, when we perform a least squares "fit" (LS fit) to the measured quantities versus the known quantities, we're assuming that only the data point's Y-location is uncertain, and it's X-location is known. Therefore, to prevent confusion over how the least squares solution is to be performed, we should plot the data of Fig. 7 with X-axis and Y-axis reversed, as in the following figure.
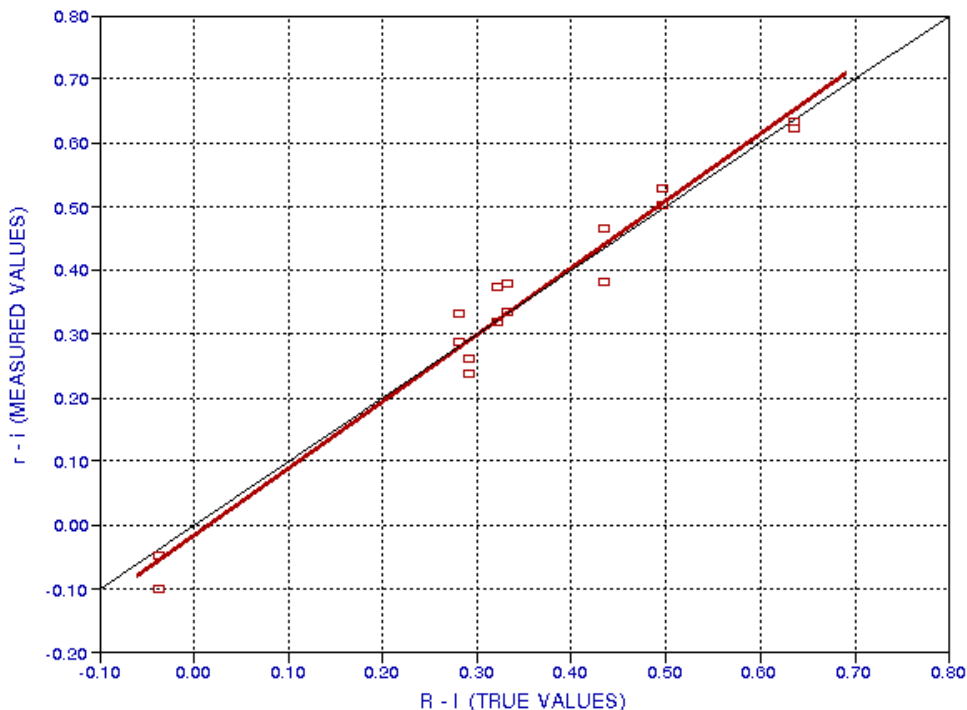
Figure 8. *Plot of measured "r - i" versus known "R - I" for a field of comparison stars (in M67). This is a proper way to display the data of the previous figure that conveys the fact that the least squares fit is to be performed with the measured "r-i" values as the dependent variable and known "R-I" values as the independent variable. The reciprocal of the LS fitted slope of the data plotted in this graph corresponds to coefficient T_{ri} (as explained in the text)*

Slightly different fitting results are obtained by treating measured quantities as the dependent variable (instead of the independent variable), and since the correct procedure is to treat measured quantities as the dependent variable the presentation of Fig. 8, with the LS fit shown, is a correct treatment of the data under question. The LS fit of measured "r-i" as dependent variable to "R-I" as independent variable produces an intercept and slope having the following meaning.

$$r-i = C_3 + C_4 * (R-I) \qquad\qquad\qquad\qquad\qquad \text{(Eqn. 2)}$$

Assume for the moment that we really want a way to calculate "R-I" from "r-i", as provided by Eqn. 1. Rearranging Eqn. 2 we are able to obtain the following relation:

$$R\text{-}I = (-C_3/C_4) + (1/C_4) * (r\text{-}i) \hspace{4cm} \text{(Eqn. 3)}$$

This has the form of a constant and slope, so let's rename terms to obtain the following.

$$R\text{-}I = C_{ri} + T_{ri} * (r\text{-}i) \hspace{4cm} \text{(Eqn. 4)}$$

where $C_{ri} = (-C_3/C_4)$ and $T_{ri} = (1/C_4)$. The term C_{ri} is called a "zero point offset" and as will be shown below it is unimportant (since it cancels out in subsequent derivations). The term T_{ri} is a transformation coefficient of first order importance. T_{ri} may be thought of as the rate of change of (R-I) with respect to (r-i), as is apparent from Fig. 7. For a "perfect system" $T_{ri} = 1$. The data in Fig. 8 produce a value for $T_{ri} = 0.951$ +/- 0.046. This value is slightly smaller than 1 (but the difference from 1.00 is not statistically significant), consistent with Fig. 7, and that implies that for the reddest stars (i.e., those have large R-I), it is necessary to correct their apparent "r-i" by attributing to them a slightly smaller redness (*i.e.*, attributing to them a R-I that is smaller than r-i).

The reader is justified in wondering why so much effort went into deriving a value for someting called T_{ri} . Be patient. We are really after an equation that permits us to convert r and i to R and I. What we've done so far is just part of our journey to understanding how this ultimate objective can be achieved.

Deriving Something Called T_r

Consider another plot showing something related to the error we appear to have in measured "r" as a function of known star color R-I.
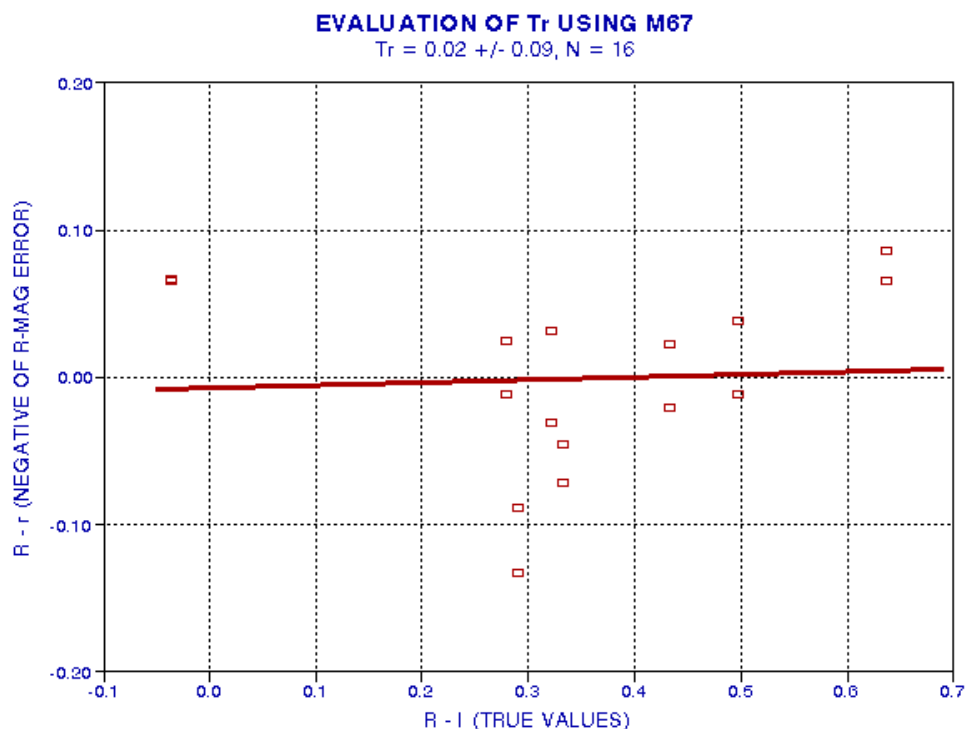
Figure 9. *The same measurements are used to show that a correction is requried to convert measured "r" to true "R" and that this correction depends on the star's R-I color. The LS fitted slope of the data graphed here corresponds to coefficient T_r.*

We notice that in this plot the measured quantity is plotted on the Y-axis, so that if error bars were used they'd be vertical. Any LS fit to this data using "R-r" as the dependent variable and "R-I" as the independent variable will be compatible with a proper treatment of observation uncertainties. In other words, our LS slope fit will provide a correct relation between the two variables. The formal LS fit has a solution:

$$(R\text{-}r) = C_r + T_r * (R\text{-}I) \hspace{4cm} \text{(Eqn. 5)}$$

where the intercept and slope have been given the names C_r and T_r. The solution values for the dat in Fig. 9 are $C_r = -0.01$ and $T_r = +0.02$ +/- 0.09. The slope term is insignificantly different from zero (value/SE = 0.22).

We've just done something without stating why we did it. We prodcued a scatter diagram that contains different information than the first scatter

diagram, with the hope that it might be useful. Now we're ready to combine equations 4 and 5 to arrive at a more useful result.

Deriving an Algorithm for Calibrating Magnitude Measurements of an Interesting Unknown Object, 2-Color Case

Consider two stars in the same CCD image, for which we shall use subscripts j and k. Assume we have calculated magnitudes (without worrying about a correct offset) that allow us to form color difference magnitudes (r-i) for each star: $(r-i)_j$ and $(r-i)_k$. Using Eqn. 4, we have:

$$(R-I)_j = T_{ri} * (r-i)_j + C_{ri}$$ (Eqn. 6)

$$(R-I)_k = T_{ri} * (r-i)_k + C_{ri}$$ (Eqn. 7)

Subtracting one from the other yields,

$$(R-I)_j - (R-I)_k = T_{ri} * [(r-i)_j - (r-i)_k]$$ (Eqn. 8)

Before making use of this difference magnitude equation, we must do something similar with Eqn. 5.

$$(R-r)_j - (R-r)_k = T_r * [(R-I)_j - (R-I)_k] + (C_r - C_r)$$ (Eqn. 9)

Suppose that star "k" is a comparison star with known magnitudes and star "j" is an object of interest, such as a supernova whose calibrated magnitude we want to determine. Let's rewrite Eqn. 9, replacing subscripts "j" and "k" with subscripts "s" and "c" (for "supernova" and "comparison star"), and also placing the subscripts with each magnitude.

$$(R_s - r_s) - (R_c - r_c) = T_r * [(R_s - I_s) - (R_c - I_c)]$$ (Eqn. 10)

and rearranging,

$$R_s = r_s + (R_c - r_c) + T_r * [(R_s - I_s) - (R_c - I_c)]$$ (Eqn. 11)

Notice in Eqn. 13 that the right side has only one "unknown" term: $(R_s - I_s)$. But "help is on the way" for notice Eqn. 8 can be written as

$$(R_s - I_s) = (R_c - I_c) + T_{ri} * [(r_s - i_s) - (r_c - i_c)]$$ (Eqn. 12)

and all terms on the right side are either known or measured. Substituting Eqn. 12 into Eqn. 11 yields

$$R_s = r_s + (R_c - r_c) + T_r * T_{ri} * [(r_s - i_s) - (r_c - i_c)]$$ (Eqn. 13)

and now all the terms on the right side are either known or measured. In other words, Eqn. 13 is a solution for the interesting object's R-magnitude, R_s. To solve for the interesting object's I-magnitude, I_s, notice the trivial relation:

$$I_s = R_s - (R_s - I_s)$$ (Eqn. 14)

All terms on the right side have been solved for above, so we also have a solution for the interesting object's I-magnitude, I_s, as well as its R-magnitude, R_s.

Here's where we stand: We've derived a procedure for determining an interesting (unknown) object's I and R magnitudes from the measurement of "i" and "r" magnitudes (without knowing offsets for either magnitude) for the interesting object and a comparison star having known (or at least "given") I and R magnitudes. In order to reduce stochastic uncertainties we can repeat this prcedure using several comparison stars, then average the several results for I_s and R_s. This is an example of using transformation equations to achieve 2-color photometry of an interesting, unknown object.

The same concepts can be used for deriving calibrated magnitudes using any two neighboring filters.

Before deriving those other filter combination solutions, let's consider what was involved in the 2-color solution. It looks like a lot of work just to arrive at two numbers, I_s and R_s, but it's really not as daunting as it seems. Once the algorithm is implemented in a spreadsheet the user merely enters measured and given numbers into labelled cells, and the answer "pops out" without effort. The hard work is in derving the algorithm, which only has to be done once. One purpose served by this "baby-step derivation" is to show that there is indeed sufficient information available to actually perform the calibration. With this knowledge there is an incentive to arrive at alternative algorithms to achieve an equivqlent, or possibly better, solution. The last section of this web page presents such an algorithm.

Algorithm for Calibrating V Measured Magnitudes

The procedure just described for calibrating measured "r" and "i" magnitudes of an interesting object (leading to R_s and I_s) can serve as a model for 2-color photometric calibration of an object's measured V-magnitude, "v". We merely substitute V and R for R and I in every instance of the previous solution (and similarly, substitute "v" and "r" for "r" and "i"). When this is done, we obtain the following (using actual measurements as illustration).
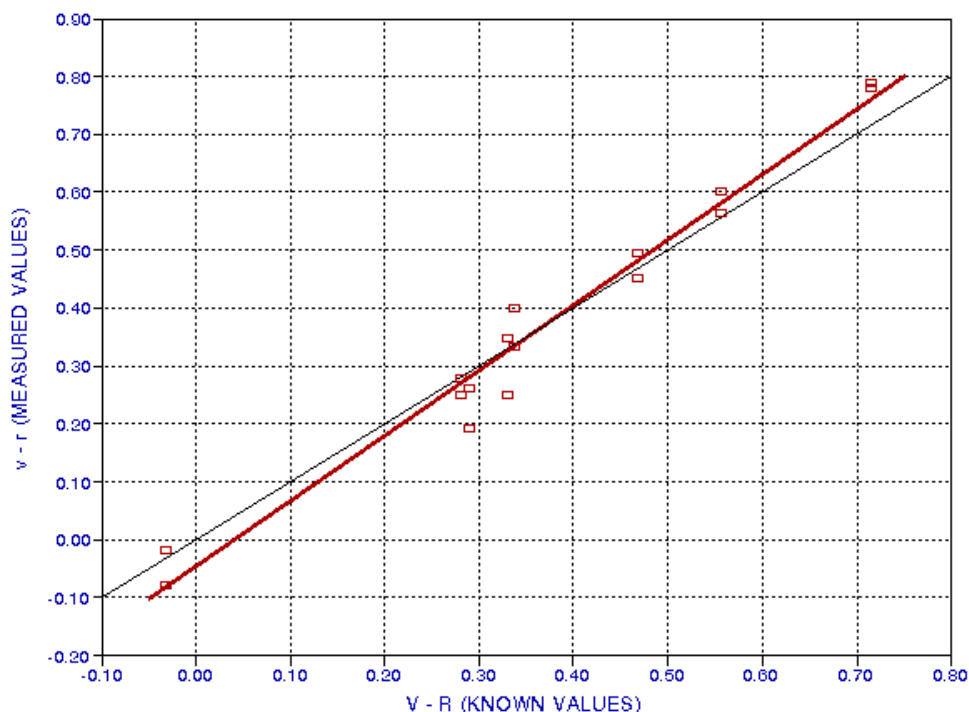
Figure 10. *Measured "v-r" versus known "V-R" for a group of comparison stars in M67. The reciprocal of the LS fit slope for this plot corresponds to coefficient T_{vr}.*

In this figure the reciprocal of the LS solution for slope corresponds to coefficient T_{vr} = 0.888 +/- 0.041. T_{vr} may be thought of as the rate of change of (V-R) with respect to (v-r).
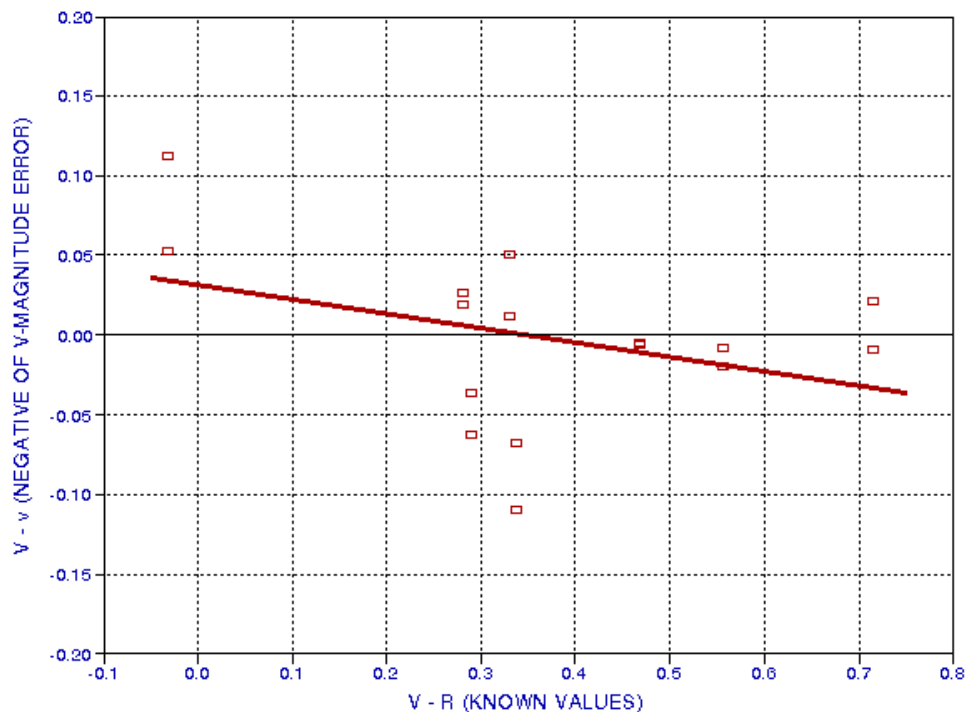
Figure 11. *Plot of "V-r" (known V magnitude minus measured "v") versus known "V-R" for a group of comparison stars in M67. The LS fitted slope corresponds to coefficient T_v.*

The data in this figure has a LS fitted slope of -0.09 +/- 0.06, which corresponds to the coefficient T_v. This solution for T_v differs is statistically compatible with zero, but it is better to use whatever the solved-for value.

Knowing T_{vr} and T_v allows for the following solution:

$$V_s = v_s + (V_c - v_c) + T_v * T_{vr} * [(v_s - r_s) - (v_c - r_c)] \qquad \text{(Eqn. 15)}$$

where all terms on the right side are either known or measured. This can be repeated for several comparisons stars, allowing for an average V_s solution. We've already solved for R_s, in the previous section, so it is not necessary to produce the counterpart to Eqns. 12 and 14. By foregoing a second solution for R_s based on a scatter plot of "R-r" versus "V-R" we are assuming that a star's I magnitude has more affect on the its V-magnitude error than the star's V-magnitude. I'm unaware of a justification for this, but it's something top keep in mind.

Algorithm for Calibrating B Measured Magnitudes

Assuming an object's measured B-magnitude is most influenced by its V-magnitude, we need to determine only the coefficient Tbv, the rate of change of (B-V) with respect to (b-v). This can be visualized using the following graph.
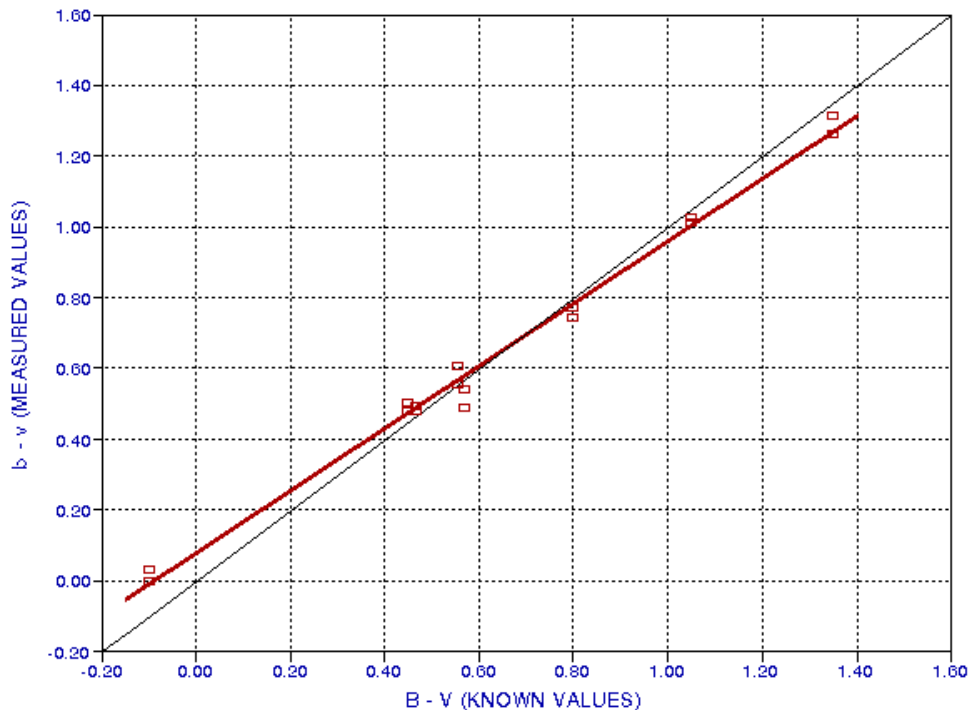
Figure 12. *Measured "b-v" versus known "B-V" for a group of comparison stars in M67. The reciprocal of the LS fit slope for this plot corresponds to coefficient T_{bv}.*

The reciprocal of the LS fitted slope of Fig. 12 corresponds to coefficient T_{bv}. Using the concepts embodied in Eqn. 12, we can solve for the term $(B_s - V_s)$ to obtain a solution for B_s.

$$B_s = V_s + (B_c - V_c) + T_{bv} * [(b_s - v_s) - (b_c - v_c)] \qquad \text{(Eqn. 16)}$$

The terms on the right side are either known or measured, so this equation affords a solution for B_s. Again, we are not taking into account a possible influence of the star's U-magnitude, and in this case there seems ample justification since few (if any) stars are brighter at wavelengths shorter than the B-magnitude filter pass band, whereas in most cases stars are brighter within the V-magnitude filter band pass than iwthin the B-magnitude band pass. (The term "brightness" is used here to refer to the number of photons coming from the star at wavelengths within the band pass under consideration, weighted by the filter's response over that band pass; since photons in the visual region dislodge no more than one electron per photon, the energy per photon is not relevant.)

Summary of Coefficient Definitions

Here's a summary of how transformation equation coefficients are determined from comparisons star observations:

    Tr = slope of (R-r) plotted versus (R-I)
    Tv = slope of (V-v) plotted versus (V-R)
    Tri = reciprocal of slope of (r-i) plotted versus (R-I)
    Tvr = reciprocal of slope of (v-r) plotted versus (V-R)
    Tbv = reciprocal of slope of (b-v) plotted versus (B-V)

Summary of Implementing CCD Transformation Equations for the BVRI Case

Assume that an image contains several comparisons stars with known values for B, V, R and I. In the equations below each comparison star's knon magnitudes are referred to below as B_c, V_c, R_c and I_c. Assume further that we have measured magnitudes b, v, r and i (with unknown offsets for each filter category) for each comparison star. These will be referred to below as b_c, v_c, r_c and i_c. Finally, we have measured magnitudes (sharing the same unknown offsets per filter category as apply to the comparisons stars). These will be referred to below as b_s, v_s, r_s, and i_s (where subscript "s" can be thought of as referring to fast-changing supernova). The task is to convert this data set to calibrated B_s, V_s, R_s and I_s for the object of interest. Here's the procedure derived above, and recommended by the AAVSO.

$(R_s - I_s) = (R_c - I_c) + T_{ri} * [(r_s - i_s) - (r_c - i_c)]$ (Eqns. 17a - 17e)

$R_s = r_s + (R_c - r_c) + T_r * [(R_s - I_s) - (R_c - I_c)]$, using the solution for $(R_s - I_s)$ in the above line

$I_s = R_s - (R_s - I_s)$, using R_s from the line above, and $(R_s - I_s)$ from the first line

$V_s = v_s + (V_c - v_c) + T_v * T_{vr} * [(v_s - r_s) - (v_c - r_c)]$

$B_s = V_s + (B_c - V_c) + T_{bv} * [(b_s - v_s) - (b_c - v_c)]$

IV. Equations for Other Filter Combinations

The foregoing assumed the observer made measurements in the all 4 colors, BVRI. Suppose an observer measures only VRI, BVR, or BV, or VR, or RI? What transformation equations should be used in those cases?

The logic of the previous section has the implicit assumption that a measurement is most affected by the objects brightness at longer wavelengths; which is to say, when correcting a magnitude measurement it is most useful to assume that the measurement using the filter at the next longer wavelength contains the information required for performing the correction. Thus, if the observations are VRI then V is most influenced by R, and R is most influenced by I. The influence of V upon R is ignored. Given this "philosophy" we can re-map the VRI situation to that of VRI treated in the previous section. The following CCD transformation equations were constructed this way (you may encounter other formulations of the same equations producing the same result).

VRI Combination

For the VRI filter combination, the following version of CCD transformation equations can be used:

$(R_s - I_s) = (R_c - I_c) + T_{ri} * [(r_s - i_s) - (r_c - i_c)]$ (Eqns. 18a - 18d)

$R_s = r_s + (R_c - r_c) + T_r * [(R_s - I_s) - (R_c - I_c)]$, using the solution for $(R_s - I_s)$ in the above line

$I_s = R_s - (R_s - I_s)$, using R_s from the line above, and $(R_s - I_s)$ from the first line

$V_s = v_s + (V_c - v_c) + T_v * T_{vr} * [(v_s - r_s) - (v_c - r_c)]$

where the definitions for T_{ri}, T_{vr}, T_r and T_v are the same as given above.

BVR Combination

For the BVR filter combination, the following version of CCD transformation equations can be used:

$(V_s - R_s) = (V_c - R_c) + T_{vr} * [(v_s - r_s) - (v_c - r_c)]$ (Eqns. 19a - 19d)

$V_s = v_s + (V_c - v_c) + T_v * [(V_s - R_s) - (V_c - R_c)]$, using the solution for $(V_s - R_s)$ in the above line

$R_s = V_s - (V_s - R_s)$, using V_s from the line above, and $(V_s - R_s)$ from the first line

$B_s = V_s + (B_c - V_c) + T_{bv} * [(b_s - v_s) - (b_c - v_c)]$

where the definitions for T_{vr}, T_{bv} and T_v are the same as given above.

RI Combination

For this combination (treated above) we may use the following:

$(R_s - I_s) = (R_c - I_c) + T_{ri} * [(r_s - i_s) - (r_c - i_c)]$ (Eqns. 20a - 20c)

$R_s = r_s + (R_c - r_c) + T_r * [(R_s - I_s) - (R_c - I_c)]$, using the solution for $(R_s - I_s)$ in the above line

$I_s = R_s - (R_s - I_s)$, using R_s from the line above, and $(R_s - I_s)$ from the first line

where the definitions for T_{ri} and T_r are the same as given above.

VR Combination

For this combination we may use the following:

$(V_s - R_s) = (V_c - R_c) + T_{vr} * [(v_s - r_s) - (v_c - r_c)]$ (Eqns. 21a - 21c)

$V_s = v_s + (V_c - v_c) + T_v * [(V_s - R_s) - (V_c - R_c)]$, using the solution for $(V_s - R_s)$ in the above line

$R_s = V_s - (V_s - R_s)$, using V_s from the line above, and $(V_s - R_s)$ from the first line

In this case the definitions for T_{vr} and T_v are the same as given above.

BV Combination

For this combination we may use the following:

$(B_s - V_s) = (B_c - V_c) + T_{bv} * [(b_s - v_s) - (b_c - v_c)]$ (Eqns. 22a - 22c)

$B_s = b_s + (B_c - b_c) + T_b * [(B_s - V_s) - (B_c - V_c)]$, using the solution for $(B_s - V_s)$ in the above line

$V_s = B_s - (B_s - V_s)$, using B_s from the line above, and $(B_s - V_s)$ from the first line

In this case the definitions for T_{bv} is the same as given above, and T_b is the LS slope fit of (B-b) versus (B-V).

V. M67 Calibration Stars

The graphs in Section III illustrating the determination of transformation equation coefficients are based on actual measurements of the M67 color calibration stars. This section describes the 13 stars with well-established BVRI magnitudes that the AAVSO recommends for use as a primary standard. It is recommended that at least once per year this set of stars, or at least a sub-set of them, be observed and used to monitor changes in the observer's system calibration. Since M67 is most conveniently observed during the winter season, two other "secondary standard" star fields can be used at other times of the year.

M67 is located in the constellation Cancer, the southern part of which is shown below.
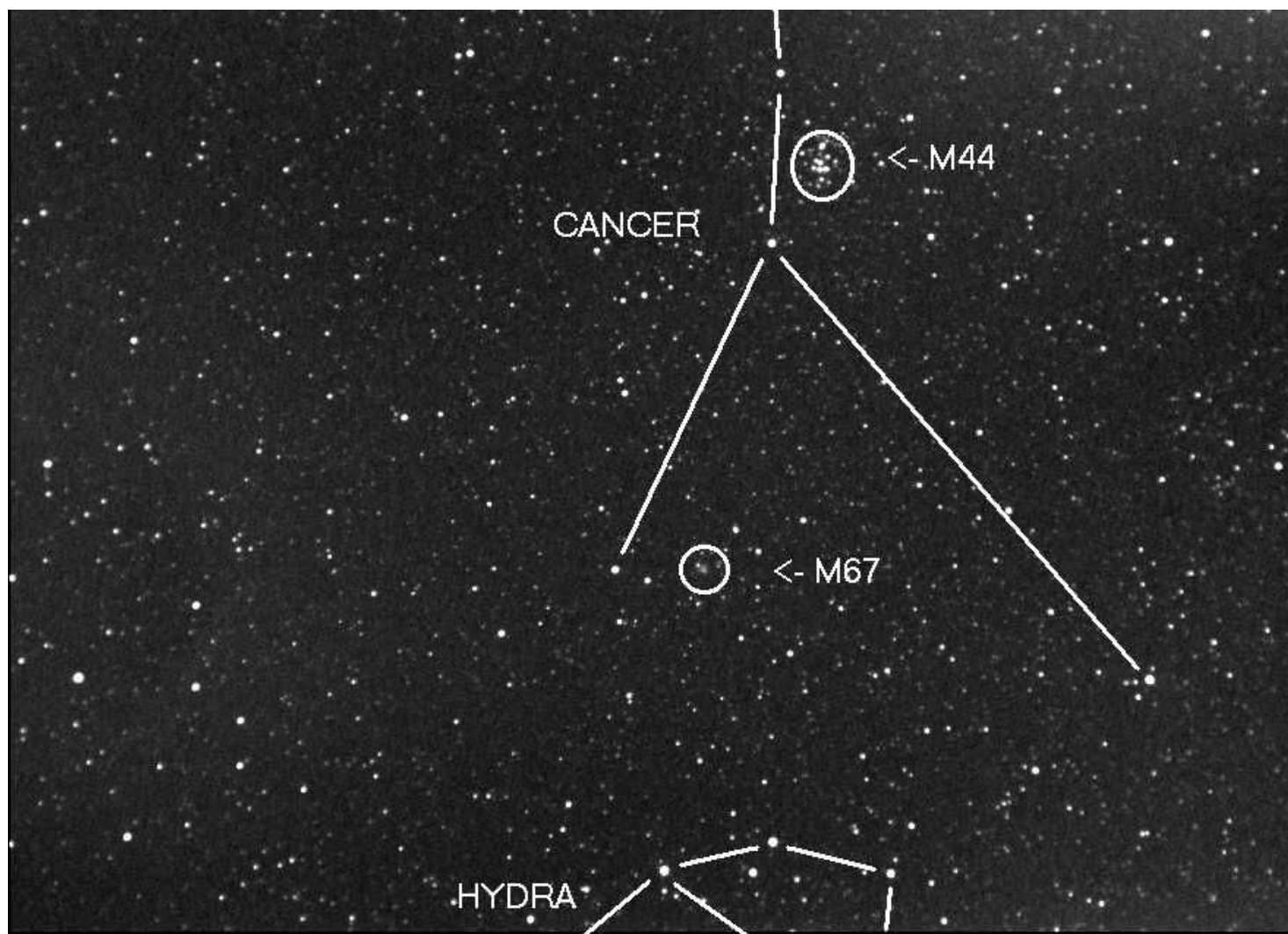
Figure 13. *Southern part of the constellation Cancer and northern part of constellation Hydra. M67 is a "smudge" at this resolution (160 "arc). The image covers an area 27.2 x 18.1 degrees.*

Figure 14. *Zoom factor 3.5. Image size is 7.8 x 5.1 degrees. The faintest stars are 13.5 magnitude. [SBIG ST-8E, Nikon 100 mm EFL, f/11, median combine of twelve 30-second exposures; 2002.05.08.3 UT, Santa Barbara residence]*
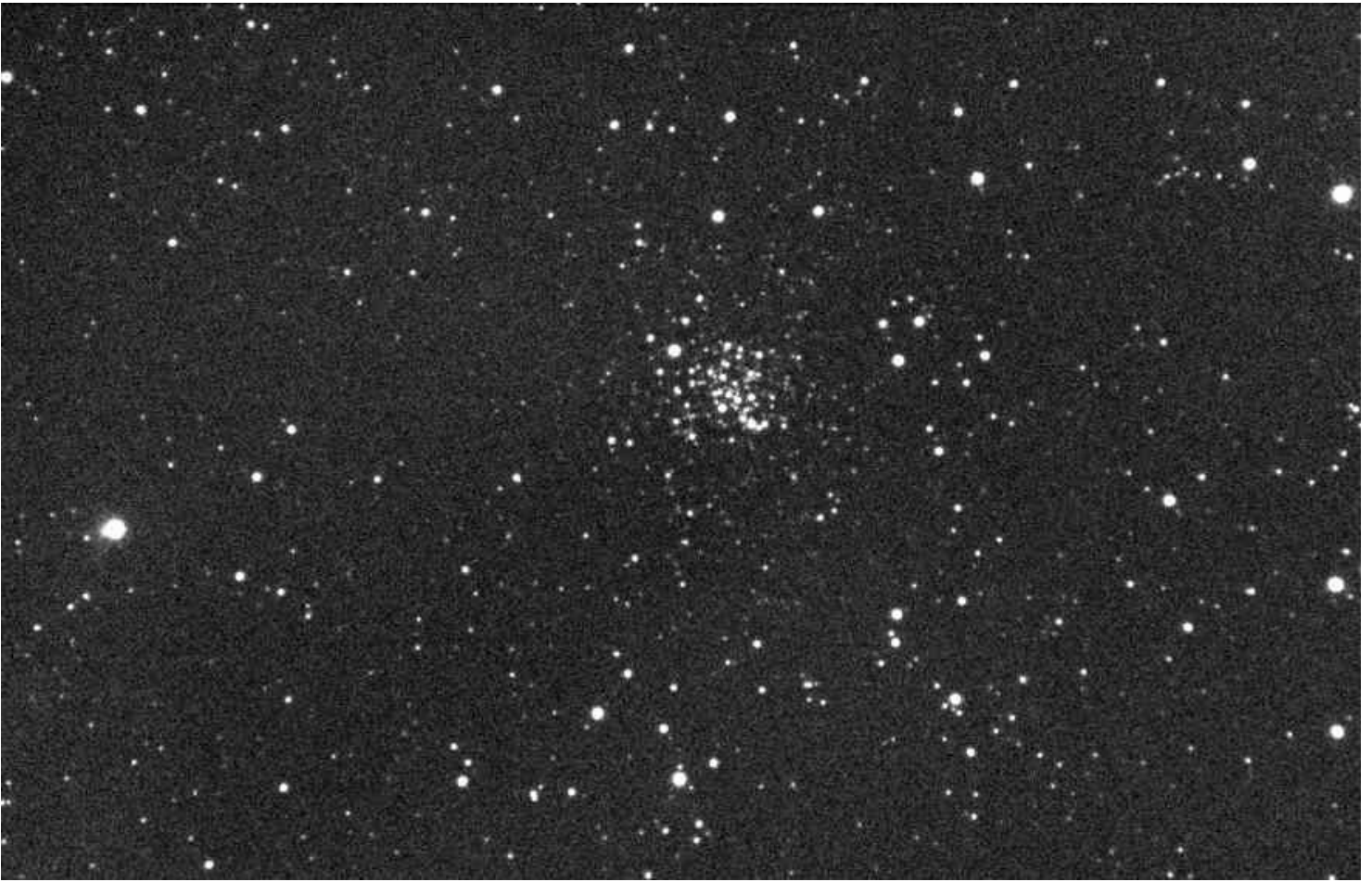
Figure 15. *Zoom factor 3. Image size is 2.59 x 1.55 degrees. The faintest stars are 13th magnitude. Star FWHM = 18"arc. [SBIG ST-8E, Nikon 300 mm EFL, f/11, median combine of ten 30-second exposures; 2002.05.08.22 UT, Santa Barbara residence]*

Figure 16. *Zoom factor 2.7 of the previous image.* RGB color image *size is 61.8 x 43.0 'arc. [Celestron CGE-1400, HyperStar, SBIG CFW-8 & ST-8XE CCD; 2003.12.10Z, Hereford, AZ.]*

Figure 17. *Zoom factor of 1.8, but using an actual telescope, showing only the M67 region, measuring 25.3 x 15.5 'arc, centered at ~8:51 and +11° 49' (resolution ~6"arc). [Meade LX200 10-inch SCT, f/6.3; SBIG ST-8E CCD, TrueTech Color Filter Wheel, Schuler BVR filters; 2002.04.22; Santa Barbara residence]*
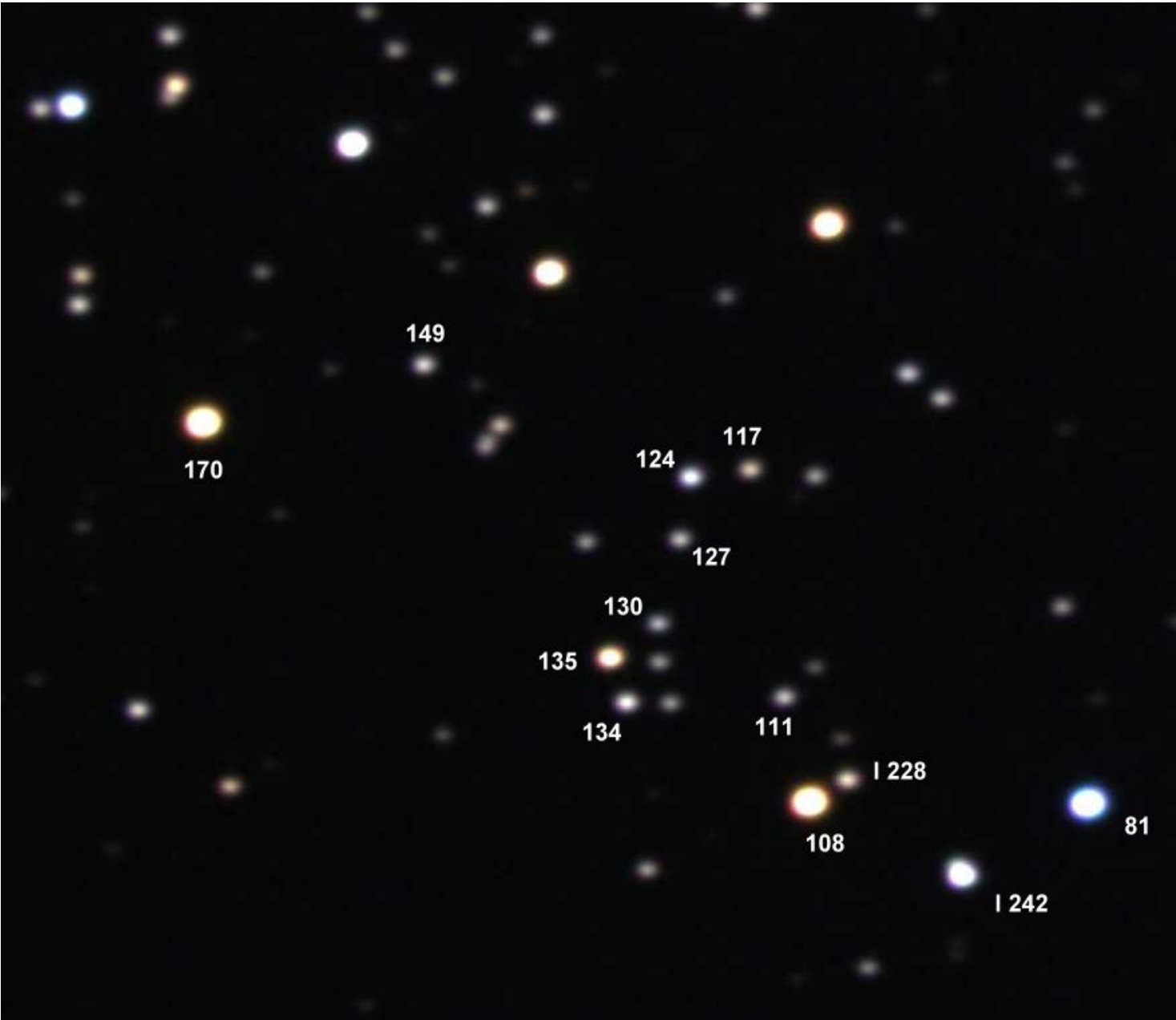
Figure 18. *Zoom of central region of the previous image. Star #135 has coordinates 8:51:22,+11° 46' 06" (epoch 2000.0). The image area is 6.82 x 4.85 'arc. The FWHM resolution is 4.0 "arc in this sum of several 1-minute unguided exposures using BVR filters. The faintest stars have a visual magnitude of about 14. Stars to be used by AAVSO observers are indicated with star numbers. The bluest star is #81 and the reddest is #108. [LX200 10-inch SCT, f/6.3, SBIG ST-8E, True Tech filter wheel w/ Schuler BVRI filters; 2002.04.22Z, Santa Barbara,CA.]*

The following table lists the M67 standard star magnitudes and colors for BVRI.

		B	V	R	I	B-V	V-R	R-I
1	170		9.663	8.961	8.336		0.702	0.625
2	149		12.550	12.208	11.877		0.342	0.331
3	117	13.430	12.630	12.163	11.729	0.800	0.467	0.434
4	124	12.584	12.118	11.838	11.558	0.466	0.280	0.280
5	127	13.322	12.769	12.439	12.118	0.553	0.330	0.321
6	130	13.318	12.869	12.580	12.289	0.449	0.289	0.291
7	135	12.487	11.436	10.880	10.383	1.051	0.556	0.497
8	134	12.825	12.256	11.919	11.587	0.569	0.337	0.332
9	111		12.730	12.402	12.076		0.328	0.326
10	108	11.052	9.701	8.986	8.350	1.351	0.715	0.636

```
11 I228        12.402 11.978 11.587      0.424  0.391
12 I242        10.884 10.616 10.351      0.268  0.265
13 81     9.929 10.027 10.059 10.095 -0.098 -0.032 -0.036
```

A subset of these stars was used for the analysis in Section III.

VI. Sample Data Analysis

In the previous section CCD transformation equation coefficients were derived from actual data made with a system consisting of a SBIG ST-8E CCD imager, Schuler photometry filters (Bu, V, Rs and Is), attached to Meade LX200 10-inch telescope (which I've also refered to as the "8E/Schuler" system). The coefficients that should be close to 1.00 (T_{ri}, T_{vr} and T_{bv}) were indeed close to the expected value, and the coefficients that were supposed to be close to zero (T_v and T_r) were indeed close to zero (this is shown in the table below). With such a system calibration corrections will be small.

In order to more easily demonstrate the need for calibrating using CCD transformation equations, I have chosen in this section to work with measurments made with the Meade 416XTE CCD and Meade color filter wheel (Model 616) blue and red filters with the Meade visual filter replaced by a good quality Optec visual filter (also referred to here as the "416XTE/616" system). This is the configuration that was NOT meant for photometry (as illustrated by Fig. 6), but which can in fact be used if care is taken with caalibration.

The following table summarizes transformation equation coefficients for the "8E/Schuler" and "416XTE/616" systems.

COEFFICIENTS FOR TWO CCD/FILTER SYSTEMS

Coefficient	"8E/Schuler" System	"416XTE/616" System
T_{ri}	0.951 +/- 0.046	n/a
T_{vr}	0.888 +/- 0.041	0.987 +/- 0.037
T_{bv}	1.136 +/- 0.029	1.89 +/- 0.13
T_r	+0.019 +/- 0.088	n/a
T_v	-0.090 +/- 0.062	-0.015 +/- 0.032

Notice that all Meade "416XTE/616" coefficients are close to the desired values except for T_{bv}. This is undoubtedly due to the filter/CCD's red-shifted spectral response (shown in Fig. 6). Incidentally, I believe in the old addage that "A measurement is not a measurement unless it comes with an uncertainty." The symbol "+/-" stands for "plus or minus" and all uncertainties quoted here are standard errors (stochastic component, only).

The sample data used to illustrate the use of CCD transformation equations are for the supernova SN2002ap (0131+15), taken near the end of the period when a light curve could be made (due to the sun's slow march toward it). The measurements were made on the date 2002.03.09 UT. Dark and flat field corrections were of course made. MaxIm DL 3.0 was used to perform "aperture photometry" for 6 comparison stars as well as the SN, all of which were within the boundaries of each image. This is a BVR system, and the measured magnitudes for this date are:

"b" = 15.37
"v" = 14.30
"r" = 13.69

Although these magnitudes are not corrected using CCD transformation equations, they do reflect MaxIm DL's process of forcing them to have an offset that minimizes the RMS differences between all comparisons star magnitudes and their known magnitudes (entered by the user). In other words, the MaxIm DL aperture photometry does a fit of all comparison star magnitudes to their known values and then produces an output of magnitudes for the comparison stars and the object of interest which do not require offset adjustments. It is my practice to record sets of magnitudes produced from several annulus choices. Each image may require carefully chosen annulus radii if the seeing was bad or if stars are near the comparison or object of interest. A spreadsheet is used to average the magnitudes from each annulus set (and inspect them for "outliers" - deserving rejection!). For the observations in question, the following figure shows how well the MaxIm DL magnitudes for the comparison stars

agreed with their "known" magnitudes.
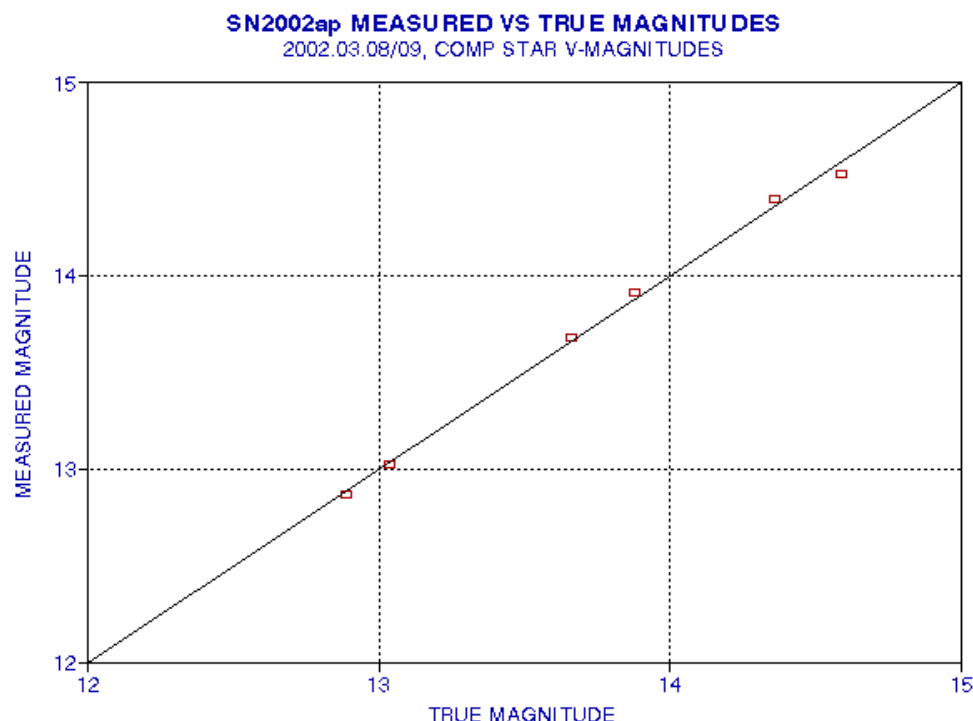
Figure 19. *Plot of "measured" versus "known" (or "true") visual magnitudes for comparison stars near object SN2002ap. RMS scatter off the 1:1 line is 0.039 magnitudes.*

Some of the differences between measured and known magnitude for the comparisons stars is due to the colors of the comparisons stars, since at this stage the CCD transformation equations have not been used. Stochastic considerations predict an uncertainty of 0.019 magnitudes. The blue filter data exhibits greater departures from the 1:1 line that cannot be accounted for stochastically, implying that the color of the comparison stars have a greater effect upon measured blue magnitude (dealt with fully in the next section).

Since this is a BVR set of data, we shall use the following CCD transformation equations:

$(V_s - R_s) = (V_c - R_c) + T_{vr} * [(v_s - r_s) - (v_c - r_c)]$ (Eqns. 23a - 23d)

$V_s = v_s + (V_c - v_c) + T_v * [(V_s - R_s) - (V_c - R_c)]$, using the solution for $(V_s - R_s)$ in the above line

$R_s = V_s - (V_s - R_s)$, using V_s from the line above, and $(V_s - R_s)$ from the first line

$B_s = V_s + (B_c - V_c) + T_{bv} * [(b_s - v_s) - (b_c - v_c)]$

The comparison star known magnitudes, and the "measured" SN2002ap magnitudes, are given in the following table:

MEASURED AND KNOWN MAGNITUDES
For Observations of 2002.03.09 UT

Star	B-Magnitude	V-Magnitude	R-Magnitude
SN2002ap	15.37	14.30	13.69
130 Meas'd	13.86	13.03	12.60
146 Meas'd	15.12	14.53	14.19
137 Meas'd	14.35	13.68	13.29
139 Meas'd	14.73	13.91	13.41
129 Meas'd	13.66	12.87	12.51
144 Meas'd	15.15	14.40	13.96
130 Known	13.84	13.04	12.59

146 Known	15.18	14.59	14.25
137 Known	14.33	13.66	13.30
139 Known	14.82	13.88	13.38
129 Known	13.60	12.89	12.50
144 Known	15.09	14.36	13.95

Notice that the top half of the table consists of measured magnitudes (lower-case magnitudes) whereas the bottom half consist of "known" comparison star magnitudes (upper-case magnitudes). Star names are based on V-magnitude; i.e., star #130 refers to the star having V = 13.0 (actually, 13.04).

Let's first only consider the use of one comparison star, "130" in the table.

Eqn. 23a states that:

$(V_s - R_s) = (V_c - R_c) + T_{vr} * [(v_s - r_s) - (v_c - r_c)]$
$(V_s - R_s) = (13.04 - 12.59) + 0.987 * [(14.30 - 13.69) - (13.03 - 12.60)]$
$\quad\quad\quad = +0.6277$

Eqn. 23b states that:

$V_s = v_s + (V_c - v_c) + T_v * [(V_s - R_s) - (V_c - R_c)]$, using the solution for $(V_s - R_s)$ in the above line
$V_s = 14.30 + (13.04 - 13.03) + (-0.015) * [(+0.628) - (13.04 - 12.59)]$
$\quad\quad = 14.3073$

Eqn. 23c states that:

$R_s = V_s - (V_s - R_s)$, using V_s from the line above, and $(V_s - R_s)$ from the first line
$R_s = 14.3073 - 0.6277$
$\quad\quad = 13.6796$

Eqn. 23d states that:

$B_s = V_s + (B_c - V_c) + T_{bv} * [(b_s - v_s) - (b_c - v_c)]$
$B_s = 14.3073 + (13.84 - 13.04) + 1.89 * [(15.37 - 14.30) - (13.86 - 13.03)]$
$\quad\quad = 15.5609$

The solution for Bs, Vs, and Rs, based on only the first comparison star, is therefore 15.561, 14.307 and 13.680.

Clearly, this is a job for a spreadsheet (or a program that reads data input files). My spreadsheet performed the above calculations for the other 5 comparison stars, producing results in the following table.

BVR SOLUTION FOR SN2002ap

Comp Star	B-Magnitude	V-Magnitude	R-Magnitude
130	15.561	14.307	13.680
146	15.853	14.356	13.750
137	15.703	14.277	13.700
139	15.681	14.268	13.660
129	15.555	14.316	13.680
144	15.592	14.257	13.680
Average =	**15.658**	**14.297**	**13.691**

The corrections in this case are dB, dV, dR = 0.29, 0.00, 0.00 magnitudes. In other words, only the blue filter magnitude needed a correction, which must be attributable to the fact that the Meade "416XTE/616" system used a blue filter that was meant for pretty pictures instead of photometry. It is valid to ask the question "When a correction of 0.29 magnitudes is required, what's the uncertainty on the correction?"

Intuitively, we should be prepared for an uncertainty as large as perhaps 0.10 magnitudes. When a filter spectral response is as far off as the one just used, it may be suggested that the the filter shouldn't be used, in the first place, or that if is used the "linear" relationships used by the CCD transformation equations may be inadequate to the task.

Alternative Transformation Procedures

The traditional CCD transformation equation suggested for use by AAVSO members is an abbreviated version of a more complete one used by the professionals. The version used by professionals includes terms that handle extinction better. Notice that the abbreviated version can produce misleading results if the "once yearly" determination of coefficients is performed with observations of the standard star field (such as M67) under unusual extinction conditions (high air mass, unusually low air mass, or unusual weather). This version can also produce misleading results even when the coefficients were evaluated from a standard star field at a typical observing situation because sometimes it is necessary to calculate differential magnitudes with images taken under unusual extinction conditions. The abbreviated equations will only be accurate when extinction is the same as occurred when the coefficients were evaluated. I have derived two alternative procedures to the abbreviated CD transformation equation calibration procedure described above and recommended for use by AAVSO members. Both alternatives are more intuitive, less prone to mistakes, and are more accurate.

The first alternative calibration procedure that appears to work for even poor filter choices is described at Alternative Calibration Procedure. I am not recommending that it be used as an alternative to the CCD transformation equation procedure (even though it is simpler and more accurate) because there's an even simpler and more accurate procedure, described in the remainder of this section.

The alternative that I now use, and which I recommend for use by other amateurs, is described in the Differential Photometry section of the following web page: All-Sky Photometry. I shall refer to it as the *Differential Photometry Iterative Procedure* (DPIP). It requires more work to set-up, but after an initial set-up using observations of a standard star field (such as M67 or a Landolt star field) it is quick, easy and does a better job of transforming unknown star images than the laborious CCD transformation procedure derived and demonstrated on this web page.

The underlying concept of the DPIP is best explained by noting that a star's observed "intensity" in a CCD image is correlated with the following factors: 1) the star's true brightness for a standard filter response (the thing we want to derive), 2) the star's color, (B-V, for example), 3) extinction associated with the filter used and the observation's air mass, 4) the observing system's spectral response when using a specific filter and observing a star with a known color (B-V, for example), 5) the dependence of a star's color on total extinction, and 6) exposure time. To the extent that there are linear relationships between all these factors it should also be possible to state that "the star's true brightness for a standard filter response" (the first item in the preceding sentence) can be derived from a linear equation involving terms for each of the items that affect the measured intensity (listed in the previous sentence). There's nothing really novel in this idea, but implementing it requires an iterative procedure which is easy when using a spreadsheet. I suspect that it has not been suggested for use as a replacement for the CCD Transformation Equations procedure because spreadsheets were not in common use until recently.

I predict that sometime this century the AAVSO will promote the use of the DPIP as an alternative to the currently suggested CCD Transformation Equation procedure because of the following advantages of DPIP: 1) it is simpler, 2) it is less prone to error (because of it's intuitive nature), 3) it is more accurate (because it allows for the facts that total extinction depends on air mass and zenith extinction can be different on different dates), 4) it is more accuate because it explicitly allows for the fact that star color varies with total extinction, and 5) once DPIP coefficients have been determiend for an observing system it is a simple matter to determine approximate magnitudes for stars when observing a star field that does not include standard reference (comparison) stars.

This web page is dedicated to: *The Pleasure of Figuring Things Out* (Thanks, Richard Feynman, for your great book titles).

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