

Ensemble Photometry Options - for AAVSO Amateur Observers Using AIP4WIN and Other Software

Tim R. Crawford

Arch Cape Observatory, 79916 W. Beach Road, Arch Cape, OR 97102
StarBoyCTX@yahoo.com

Preface 3/30/06

The reader should be aware that this was originally prepared for the JAAVSO. Both the original and the revised submissions were rejected as not meeting the necessary standards of the referee and or the editor; the specific methodology of the three individual options presented does not appear to have been at issue. I have attempted with this third revision to address the most recent key issues raised by the referee, as best I can, and self-publish.

Like the majority of AAVSO observers I am simply an amateur astronomer and have no formal training. This effort, a first attempt at a “scientific” paper, is the result of my own frustration at locating “understandable” information on ensemble photometry; specifically, what I term the “Master Star” option. After a number of months of research I felt compelled to share with others what I had learned, with the hope that their own efforts would be less frustrating. Time might have been better served if I had simply approached this subject initially as a “cookbook” offering and immediately self-published. I hope that the observer will find portions of this to be beneficial for their own purposes and encourage others to continue developing this “topic” for AAVSO members.

It was suggested to me by one AAVSO member that maybe this should become a “living” document. I will keep that in mind for the future and would certainly welcome others to add to this document if they have an interest.

Lastly, corrections are both encouraged and welcomed.

Abstract

A presentation of three optional methods, *Mean Value*, *Weighted Average* and *Master Star*, for an ensemble photometric solution, using differential aperture photometry and AAVSO charts, for the typical AAVSO amateur variable star observer. While this presentation uses the CCD image analysis data output from AIP4WIN V2 the methods presented should be able to be used by any photometry program that the typical AAVSO observer might be using. None of the methods presented are new; this document simply brings them together, with examples, in a simple understandable presentation. A brief introduction to “traditional” differential aperture photometry as used by typical AAVSO amateur observers is also included in the Introduction.

1. Introduction

AIP4WIN V2 is the software accompanying the second edition of “*The Handbook of Astronomical Image Processing.*” (Berry and Burnell, 2005)

Photometry is the measurement of a star’s brightness and the magnitude system is expressed using Pogson’s Formula:

$$m_1 - m_2 = -2.5 \log(F_1/F_2) \quad (1)$$

This states that the difference between the magnitudes of two stars, $m_1 - m_2$, is dependent upon the ratio of their fluxes F_1 and F_2 (see equations 10 and 11). (Berry and Burnell, 2005)

Differential photometry is the comparison of stars, within the same image (ideally of the same color and constant output, but in reality not a common occurrence), which have known magnitude values with the target star.

“Traditional” amateur differential photometry determines the final magnitude of the variable star (V) being studied through the use of a comp star (C), with a known magnitude, and a check star (K), with a known magnitude and looks like this (also see equation 3):

$$V - C^* = v - c \quad (2)$$

*Provided the known $K - C = \text{observed } k - c$ (AAVSO, 2004)

Where:

c = instrumental magnitude of comp star

C = known magnitude value of a comp star (from AAVSO or similar chart)

k = instrumental magnitude of a check star

K = known magnitude value of a check star (from same chart as “C” star)

v = instrumental magnitude of variable star

V = magnitude of the variable star

In reality $K - C$ seldom actually is a precise equal to the observed $k - c$ in that typically it is not unusual to note differences of several hundredths. The challenge for the observer to match the known values to the measured values as closely as possible.

Amateur variable star observers depend upon C and K magnitude values from charts created for that purpose by individuals or organizations, such as the AAVSO (www.AAVSO.org). For additional information on the topic of C , K and V star differential aperture photometry see Howell, etal, 1988, Berry and Burnell, 2000 and the AAVSO CCD manual (AAVSO, 2004)

Software is used to process image data using overlaid aperture rings. The observer chooses the radius of software generated annulus and aperture rings to overlay on top of selected comp stars (C) and the target star. The software will analyze the CCD image and generate raw instrumental magnitudes for target star (v), the comp star (c) as well as for the check star (k).

Once the program has computed the raw instrumental magnitudes we can then use those values to compute the final magnitude (V) of the variable star from equation 2:

$$V = (v - c) + C \quad (3)$$

Ensemble photometry uses multiple comp stars (stars with known magnitude values) instead of a single comp star to arrive at the final magnitude value.

With an ensemble, any errors in the known values of the comp stars vs. the measured values, where $K-C$ does not equal $k-c$ (or C_2-C_1 does not equal c_2-c_1 -see paragraph after next), will average out.

AIP4WIN V2 not only computes raw instrumental magnitudes but allows the observer to uniquely (as opposed to many other photometry programs) zero point those instrumental magnitudes, by adjusting the Zconstant (see equation 10) within the software, prior to the final data output such that the $C = c$ on for at least one comp star on the image (known comp star value equals the measured instrumental magnitude). Once a single comp star has been zero pointed the remaining comp star choices are easy to verify by insuring that any differences between known and measured values are minimal.

Since the traditional check star (K) is simply another comp star we can alter the condition (having known $K-C = \text{observed } k-c$) to having the known comp values = the observed values, that is: $C_2-C_1 = c_2-c_1$. With the ability to readily generate instrumental magnitudes for multiple comp stars Version 2 of AIP4WIN has eliminated the concept of a single star using the designation of a “K” (check) star as multiple comparisons can be made between the multiple comp stars.

Version 2 of AIP4WIN also uniquely (as opposed to some of the other photometry software programs used by amateur observers) computes the sigma (a.k.a. uncertainty or error) for each instrumental magnitude in a manner, which does not rely upon simple Poisson statistics (noise proportional to the square root of the signal). See appendix for the sigma formula used by this software.

Additionally, AIP4WIN version 2 uniquely (again, as opposed to some of the other photometry software programs used by amateur observers) allows the observer to have the software compute a single ensemble instrumental magnitude value for all of the comp stars selected as well as the sigma value for that ensemble.

Three different options, using AIP4WIN V2, for the photometric solution of an ensemble of stars, will be presented. Please understand that the three options presented should be able to be used with the output of any photometry program that the typical AAVSO observer might be using.

The first method I will call the *Mean Value* approach; the second method will be called the *Weighted Average* approach while the third method will call the *Master Star* approach. The names of these three particular methods are used herein by the author (without claim to their originality), in the absence of literature to the contrary, in an attempt to differentiate them from one another.

For simplicity, I have chosen to use only three comp stars in presenting each ensemble option, although AIP4WIN V2, for all practical purposes, allows for an almost unlimited number of comp stars.

The example used with each option will use the same instrumental magnitudes and sigma values, as output by AIP4WIN V2, as they first appear in Table 1.

2. Mean Value Option for Ensemble Photometry

This option (equation 4) appears to be the choice of most amateur observers, using multiple comp stars as well as many professional astronomers.

By solving for V for a number of comp stars (see equation 3) we can create an ensemble of values for which a mean value magnitude, V_m , is calculated, as shown in equation (4). (Henden and Kaitchuck, 1982)

$$V_m = 1/N \sum_{i=1}^n V_i \quad (4)$$

The Summation sign (\sum) used in equation (4) simply means the sum of V values from $i = 1$ to N (total number of data points).

Here are CCDV instrumental magnitude values for the Variable Star 3C 66A from 10/27/05, UT 04:11:50 (author) using three comp stars and AIP4WIN V2 for the photometric solution (instrumental magnitudes and sigma).

Comp star values (a.k.a. sequence values) are from the AAVSO "E" chart for 3C 66A. CCDV denotes the use of a visual or V filter.

Star	Comp Value (C)	Instrumental Mag *	Sigma (σ)
3C 66A	na	15.414 (v)	.0120($v\sigma$)
Comp 1	11.800 (C_1)	11.775 (c_1)	.0010 (σ_1)
Comp 2	12.200 (C_2)	12.220 (c_2)	.0012 (σ_2)
Comp 3	13.100 (C_3)	13.114 (c_3)	.0021 (σ_3)

Table 1.

Solve for V values using equation (3) and values from Table 1:

Star	$v-c_x$	Comp Magnitudes	$V_x = (v-c)+C$
Comp 1	15.414-11.775 = 3.639	11.800	15.439
Comp 2	15.414-12.220 = 3.194	12.200	15.394
Comp 3	15.414-13.114 = 2.300	13.100	15.400

Table 2.

Solve for V_m using equation (4):

$n = 3$ because we have three data points in this example.
 $V_1 = 15.439$, $V_2 = 15.394$, $V_3 = 15.400$

From Table 1.
 From Table 2.

$$V_m = 1/3 \sum_{i=1}^3 V_i$$

Replacing n with 3

$$V_m = 1/3 (V_1+V_2+V_3)$$

$$V_m = 1/3 (15.439+15.394+15.400)$$

Expanding expression
 Substituting for V_x

$$V_m = 15.411$$

Solved

Calculate the error of the magnitude solved by the *Mean Value* approach where $v\sigma$ = the sigma of the variable star and σ_x = the sigma of the individual comp stars. The symbol (^) indicates that what follows is an exponential expression.

$$\text{Error} = \text{SQRT}((v\sigma^2) + (1/n \sum_{i=1}^n \sigma_i^2)) \quad (5)$$

$n = 3$ as we have three data points in this example (σ_i)

From Table 1

$$\sigma_1 = .0010, \sigma_2 = .0012, \sigma_3 = .0021; v\sigma^{\wedge} = .0120$$

From Table 1

$$\text{Error} = \text{SQRT}((v\sigma^2 + (1/3 \sum_{i=1}^3 \sigma_i^2)))$$

Replacing n with 3

$$\text{Error} = \text{SQRT}((.0120^2) + ((.0010 + .0012 + .0021)/3)^2)$$

Substitute for σ_i & $v\sigma$

$$\text{Error} = \text{SQRT}(.000144 + .000002054)$$

Simplify expression

$$\text{Error} = \text{SQRT}(.000146054)$$

Simplify expression

$$\text{Error} = .0121 \text{ [magnitude]}$$

Solved

3. *Weighted Average Option for Ensemble Photometry*

While Richard Berry suggested this method to me, it apparently has been previously published in PASP by both Honeycutt and Everett (separately), although I do not have the specific references.

The *Mean Value* option gives equal value to each of the V_x solutions when, in reality, they are seldom equal, simply because of the sigma statistics. A more statistically significant solution would be to weight the average by the inverse of the standard errors, where σ_x = individual sigma/error and V_i = the individual solved V magnitudes. (Berry 2005,1)

$$V_w = \sum_{i=1}^n (V_i/\sigma_i)/(1/\sigma_i) \quad (6)$$

Solve for V_w using equation (6) where V_w = Weighted Average Magnitude:

$n = 3$ because we have three data points in this example (V_x) Using Table 2 data

$$V_1 = 15.439, V_2 = 15.394, V_3 = 15.400$$

From Table 2

$$\sigma_1 = .0010, \sigma_2 = .0012, \sigma_3 = .0021$$

From Table 1

$$V_w = \sum_{i=1}^3 (V_i/\sigma_i)/(1/\sigma_i)$$

Replacing n with 3

$$V_w = (V_1/\sigma_1 + V_2/\sigma_2 + V_3/\sigma_3)/(1/\sigma_1 + 1/\sigma_2 + 1/\sigma_3)$$

Expand expression

$$V_w = (15.439/.0010 + 15.394/.0012 + 15.400/.0021)/1/.0010 + 1/.0012 + 1/.0021) \text{ Substitute}$$

$V_w = (15,439+12,828+7,333.33)/(1000+833.33+476.190)$	Simplify expression
$V_w = (35,600.33)/2309.52$	Simplify expression
$V_w = 15.415$	Solved

The error for this option may be calculated using the same equation as used for the *Mean Value* option, equation (5).

Note: Of interest for those wishing to explore other “weighting” options is a discussion by Henden and Kaitchuck regarding a concept they called the “*weighted mean*.” (Henden and Kaitchuck, 1982)

4. *Master Star Option*

While some professional astronomers, as well as a few amateurs, use this approach (see Deeg and Laurance, 2001), I have not been able to locate any previous literature directed at the amateur astronomer using this approach (excepting AIP4WIN V2).

The *Master Star* option involves the creation of a single composite comp star from the sum of the fluxes of all the comp stars to produce a single instrumental magnitude for the ensemble (ens) (Henden 2003).

This value (ens) is then subtracted from the instrumental magnitude of the variable star (v) and added to the single Master Star value (ensemble) created from the Comp values (C_{ens}) as shown by equation (7). V_{ms} = Master Star option magnitude.

$$V_{ms} = (v - c_{ens}) + C_{ens} \tag{7}$$

Equation (7) is really the same as equation (3) except that “Ens” represents the magnitude of the combined fluxes

AIP4WIN V2, in addition to the instrumental value for each comp star, computes an ensemble “master star” Instrumental magnitude value for the whole of the ensemble as well as the $v - c_{ens}$ value.

This leaves the observer with only the need to calculate the “master star” comp magnitude value (C_{ens}) when using AIP4WIN V2 (equation (8) can also be applied to the instrumental values if your software does not accomplish this for you).

“What we need to do is to convert each of the comp magnitudes back to a flux value then sum the individual fluxes and convert them back to magnitudes.” (Berry 2005,2) See the Appendix for the derivation of the following formula, equation (8)

$$C_{ens} = (-2.5) \text{Log}10 \sum_{i=1}^n 10^{(-C_i/2.5)} \tag{8}$$

Where C_i equals the individual comp values.

The below data is from the same source and observation as Table 1 with the addition of the ensemble instrumental magnitude which value was output by AIP4WIN V2 along with the rest of the data (Table 1), including the V-ens value.

Star	Comp Value	Instrumental Mag	Sigma	v- c _{ens}
3C 66A		15.414 (v)	.0120 (vσ)	
Comp 1	11.800 (C ₁)	11.775 (c ₁)	.0010 (σ ₁)	
Comp 2	12.200 (C ₂)	12.220 (c ₂)	.0012 (σ ₂)	
Comp 3	13.100 (C ₃)	13.114 (c ₃)	.0021 (σ ₃)	
Ensemble		11.047 (c _{ens})	.0007 (σ _{ens})	
v- c _{ens}		15.414(v)-11.047 (c _{ens})	.0139 (σ _{vms})	4.367

Table 3.

Solve for the comp ensemble value (C_{ens}) using equation (8). A spreadsheet ready formula is listed in the Appendix, equation (12).

N = 3 as we have three data points in this example

$$C_1 = 11.800, C_2 = 12.200, C_3 = 13.100$$

From Table 3

$$C_{ens} = (-2.5)\text{Log}10 \sum_{i=1}^3 10^{(-C_i/2.5)}$$

Replacing n with 3

$$C_{ens} = (-2.5)\text{Log}10(10^{(-C_1/2.5)} + 10^{(-C_2/2.5)} + 10^{(-C_3/2.5)})$$

$$C_{ens} = (-2.5)\text{Log}10(10^{(-11.8/2.5)} + 10^{(-12.2/2.5)} + 10^{(-13.1/2.5)})$$

$$C_{ens} = (-2.5)\text{Log}10(10^{-4.72} + 10^{-4.88} + 10^{-5.24})$$

$$C_{ens} = (-2.5)\text{Log}10(1/10^{4.72} + 1/10^{4.88} + 1/10^{5.24})$$

$$C_{ens} = (-2.5)\text{Log}10(1/52480.746 + 1/75857.757 + 1/173780.08)$$

$$C_{ens} = (-2.5)\text{Log}10(.0000379916)$$

$$C_{ens} = 11.051$$

To solve for the ensemble variable star magnitude using equation (7):

$$V_{ms} = (v - c_{ens}) + C_{ens}$$

$$V_{ms} = 4.367 + C_{ens}$$

$$V_{ms} = 4.367 + 11.051$$

$$V_{ms} = 15.418$$

Replace (v-c_{ens}) from Table 3

Replace C_{ens} from Eq. 8 Solution

Solved

In addition to providing the sigma for each individual instrumental magnitude AIP4WIN V2 also calculates the *Master Star* ensemble sigma value as well as the v- c_{ens} sigma value (see last two rows of the Sigma column in Table3).

v- c_{ens} calculation:

$$\text{Error} = \text{Sqrt}((v\sigma)^2 + (\sigma_{ens})^2) \tag{9}$$

$$\text{Error} = \text{Sqrt}((.0120^2) + (.007)^2)$$

$$\text{Error} = \text{Sqrt}(.000193)$$

$$\text{Error} = .0139 \text{ [magnitude]}$$

Replace with values

Simplify expression

Solved

The reader should understand that equation (8) may be used to sum the values of either the comp stars or their instrumental magnitudes. In the case of AIP4WIN V2, the software sums the instrumental magnitudes leaving the user to deal only with the summing of the comp star values. Users of other programs would have to then use this

same equation to sum their own derived instrumental values as well as the comp values to use the “Master Star” approach.

5. Analysis

The final V magnitude from each option shows that each of the options provide reasonable results with the same specific set of data (Table 1) having a maximum difference of only .007 magnitude between them.

Mean Value: 15.411
Weighted Average: 15.415
Master Star: 15.418

In this instance the comp $C_x - c_x$ values are close to zero, that is the known values are pretty close to being equal to the measured values (see Table 1).

Now lets examine a case where at least one $C_x - c_x$ value has a greater difference.

Star	Comp Mag (C)	Instrmntl Mag (c)	Sigma (σ)	V Mag Mean Option	V Mag Weighted Option	V Mag Master Star Option
SN 2005CS		14.645	.0086	14.546	14.582	14.584
Comp 1	13.500	13.556	.0037			
Comp 2	13.700	13.730	.0039			
Comp 3	15.100	15.310	.0146			
Ensemble	12.715	12.776	.0028			

Table 4

The final V magnitude from each option, in Table 4, shows some significant differences showing a maximum difference of .038 magnitudes (Vms vs Vm)

Mean Value: 14.546
Weighted Average: 14.582
Master Star: 14.584

In this instance all the comp, $C_x - c_x$, values are not close to zero; specifically comp three shows a $C_3 - c_3$ difference of some .21 magnitudes.

Now lets examine the same data presented in Table 4 but dropping Comp 3 data as it was a poor choice for a comp star; a good reminder to always LOOK at the DATA.

Star	Comp Mag (C)	Instrmntl Mag (c)	Sigma (σ)	V Mag Mean Option	V Mag Weighted Option	V Mag Master Star Option
SN 2005CS		14.645	.0086	14.602	14.602	14.601
Comp 1	13.500	13.556	.0037			
Comp 2	13.700	13.730	.0039			

Table 5

The final V magnitude from each option for Table 5 shows that each of the options provide reasonable results, with only a .001 magnitude difference between them.

Mean Value: 14.602
Weighted Average: 14.602

Master Star: 14.601

In this instance the comp $C_x - c_x$ values are much closer to zero.

Comparing Table 4 & 5 Final V magnitude values:

	From Table 4	From Table 5	Difference
<i>Mean Value:</i>	14.546	14.602	.056 magnitudes
<i>Weighted Average:</i>	14.582	14.602	.020 magnitudes
<i>Master Star:</i>	14.584	14.601	.017 magnitudes

When we compare the solutions from Table 4 & 5 it should be noted that the *Weighted Average* and the *Master Star* options produced answers much closer to the answer where the poor comp choice was omitted (from Table 5) while the *Mean Value* has a much larger difference.

6. Summary

The Final V magnitudes from the three ensemble options, using Table 1 data, clearly show that any one of the options provides acceptable data, in that the differences are not significant.

However, when we have the instance where the known comp value significantly varies from the measured value as shown by comp 3 in Table 4, the *Mean Value* option provides an answer .036 magnitudes less than the *Weighted Average* option and .038 magnitudes less than the *Master Star* option. When we omit the comp 3 star from the computations, as shown in Table 5, we discover that all three-ensemble options provide acceptable data, in that the resulting differences of the answers are not significant. In addition when we compare the Final V magnitude results from the data in both Tables 4 and 5 it is clear that the both the *Weighted Average* and *Master Star* options produced values that were much closer together both with and without the poor comp 3 star.

This demonstrates to me that while the *Mean Value* option can provide excellent results with accurate comp values (when the known comp star values are close to the measured comp star values) but when poor comp stars are present within the ensemble the *Mean Value* answer deteriorates.

While the *Weighted Average* and *Master Star* options also show some deterioration when a poor comp choice is included within the ensemble, they also show that their values are more significant than the *Mean Value* option.

Arne Henden stated to me that he uses the *Mean Value* method with good reason (Henden 2005):

“When you convert to flux, the brightest star in the ensemble dominates completely. So any error in its measure directly impacts the target star. By using the magnitude averaging method, you give all stars in that window equal weight, yielding a more robust answer. As long as the individual values do not stray greatly from the mean, this is equivalent to using fluxes.”

As both Mr. Henden and I have pointed out, the use of the *Mean Value* option requires good comp values; that is when the known value pretty much equals the measured value.

Amateurs sometimes use charts having some stars with “poor” comp values within the field of view. I believe that this reason is adequate justification for the consideration of either the *Weighted Average* or the *Master Star* Option.

If the observer prefers to stay with or adopt the *Mean Value* ensemble option then it is important, if you have AIP4WIN V2, that the zero point tool be used to aid in the examination of comp values so that a reasonable equality is maintained between the known comp star magnitudes (C) and the measured comp star magnitudes (c); if you do not use AIP4WIN then you must examine your final values to accomplish the same result.

I would encourage observers to “play” with the three methods and to reach your own conclusions.

Finally, of overwhelming major importance, regardless of the methodology used, is the need to always LOOK at the DATA.

Appendix

Derivation of equation (8):

AIP4WIN measures four elements from the CCD image and selected tools to generate raw instrumental magnitudes: Total aperture counts (flux of star), total aperture pixels (size of aperture in pixels), total annulus counts (sky flux) and total annulus pixels. All the photometric solutions then flow from just these four software-measured numbers. (Berry and Burnell, 2005)

$$\text{Star_Magnitude} = Z_{\text{constant}} - 2.5 * \log_{10}(\text{Star_ADUs} / \text{Integration_Time}) \quad (10)$$

Z_{constant} = An arbitrary constant in the equation for adjusting the raw instrumental magnitude so that its value will equal a known comp star value within the same image. Note that Star_Magnitude is a raw instrumental magnitude, uncorrected for color or extinction.

Where:

$$\text{Star_ADUs} = \text{starpixels} * ((\text{Aperture_ADUs} / \text{Aperture_Pixels}) - (\text{Annulus_ADUs} / \text{Annulus_Pixels})). \quad (11)$$

Where:

Aperture_Pixels = the number of pixels in the star diaphragm.

Aperture_ADUs = the total pixel value found in the star diaphragm

Annulus_Pixels = the number of qualified pixels in the sky annulus

Annulus_ADUs = the total pixel value in qualified pixels in the sky annulus

(ADU's = Analog-to-digital units and is an expression of the photon/flux values measured from the CCD image.)

Using equation (10) solve for equation (8):

Z_{constant} = an arbitrary zero point and will now be dropped.

ADU/Time = $10^{(-mag)/2.5}$
 Magnitude = $-2.5\log_{10}(10^{(-mag)/2.5})$
 Therefore with an ensemble

Solve for ADU/Time
 Substitute ADU/Time

$$C_{ens} = (-2.5)\log_{10}\sum_{i=1}^n 10^{(-C_i/2.5)}$$

Q.E.D.

As an aid to AAVSO observers, here is a spreadsheet “ready” formula, using three stars, for an ensemble “Master Star” of either the instrumental magnitudes or the comp magnitudes; this should be easy to expand for additional stars and can be used with any photometry software that produces instrumental magnitudes (c) in addition to the chart comp star values (C).

$$=-2.5*\text{LOG}_{10}(\text{SUM}(10^{(-C_1/2.5)},10^{(-C_2/2.5)},10^{(-C_3/2.5)})) \quad (12)$$

Where C_x = the comp magnitude of an individual star

It was not my intent to deal with the issue of “error” within this paper nor have I done so, however, I am including the equation for the sigma values (a.k.a. error or uncertainty) used by AIP4WIN V2 so that the reader will understand how much more sophisticated this approach is to using the SQRT (1/SNR) or similar derivations.

Derivation of the instrumental magnitude sigma values (σ) listed in the tables and expressed in equation (14):

Equations (13) and (14) are taken directly from the help file for the program AIP4WIN V2. For further discussion on this topic see: Berry and Burnell, 2005 and Howell, 2000.

“This procedure assumes Poisson statistics for electrons and a Gaussian deviate for readout noise. The prefix "N" indicates that the units are electrons.” (Berry and Burnell, 2005)

Where:

$$\begin{aligned} N_{\text{StarMinusSky}} &= \text{Gain} * \text{Star_ADUs} \\ N_{\text{Sky}} &= \text{Gain} * (\text{Annulus_ADUs} / \text{Annulus_Pixels}) \\ N_{\text{Dark}} &= \text{Dark_Current} \\ N_{\text{Ron}} &= \text{Readout_Noise} \\ \text{PixRatio} &= 1 + (\text{Aperture_Pixels} / \text{Annulus_Pixels}) \\ N_{\text{Quant}} &= 0.289 * \text{Gain} * \text{Gain} = \text{quantization noise} \end{aligned}$$

$$\text{Noise} = \text{Sqr}(N_{\text{StarMinusSky}} + \text{Aperture}(\text{pixels}) * \text{PixRatio} * (N_{\text{Sky}} + N_{\text{Dark}} + N_{\text{Ron}} + N_{\text{Quant}})) \quad (13)$$

$$\text{Star_StdDev} = 1.0857 * (N_{\text{StarMinusSky}} / \text{Noise}) [\text{magnitudes}] \quad (14)$$

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